

Gamma Ray Spectroscopy and Counting Statistics

August 21, 2002

In this experiment we will study the interaction of gamma radiation with matter. The write-up will briefly summarize the subjects we covered in our introductory lecture. You will measure and interpret the spectrum of gamma ray energy losses, resulting from interaction with sodium iodide (NaI) and plastic scintillation detectors.

This description also introduces the essentials of data taking in the lab using the program **PCAI**. Data visualization and statistical analysis will be done on the UNIX workstation of the HEP group using the CERN software package PAW which has developed into a standard tool in the nuclear and particle physics research community.

Contents

1	Tasks	3
2	Introduction	3
2.1	Photoelectric Effect	3
2.2	Compton Scattering	5
2.3	Pair Production	8
2.4	Total Absorption	10
3	The used Equipment	16
3.1	The Observed Spectra	16
3.2	Data Taking	21
3.3	NaI Detector	22
4	Tasks for the first day	23
4.1	Get started	23
4.2	Counting Statistics	23
5	Tasks for the second day	24
5.1	Energy Calibration	24
5.2	Energy resolution	25
6	Tasks for the third day	25
6.1	Energy Calibration	25
6.2	Energy resolution	26
7	Data Transfer via the Computer Network	27
8	Plotting and Data Analysis with PAW	27
8.1	Data Analysis in PAW	30
8.2	Analysis Tasks	32
8.2.1	NaI Data	32
8.2.2	Plastic Scintillator Data	32
9	Appendix A: χ^2 Fitting	32

1 Tasks

1. Read and understand the experiment description.
2. Solve the pre-lab problems given in the write-up. You will have to turn in the written solutions before you can work on the experiment.
3. First day: experimentally demonstrate that mean and variance stand in a well defined relation in a counting experiment.
4. Second day: perform an energy calibration for the NaI detector. Determine the energy required to create a photo electron for this detector type.
5. Third day: perform an energy calibration for the plastic scintillation detector. Determine the energy required to create a photo electron.
6. Prepare a written report summarizing your work and results.

2 Introduction

In this experiment we will study how energetic photons, with energies higher than 0.1 MeV, interact with matter. We will do so by using two different types of scintillation detectors. We will measure and interpret the spectra of energy losses. This will give us insight into some basic principles of radiation detection.

Photons are neutral particles which cannot be easily detected themselves. In order to measure their energy we have to make use fundamental reactions which fully or partially transfer the photon energy onto an electron. The kinetic energy of electrons can then be measured using different detection devices. In the following we will briefly summarize these interactions and outline what detector response to expect. A more detailed discussion can be found in *W.R. Leo chapter 2*.

2.1 Photoelectric Effect

The photoelectric effect involves the absorption of a photon by an atomic electron with the subsequent ejection of the electron from the atom, leaving

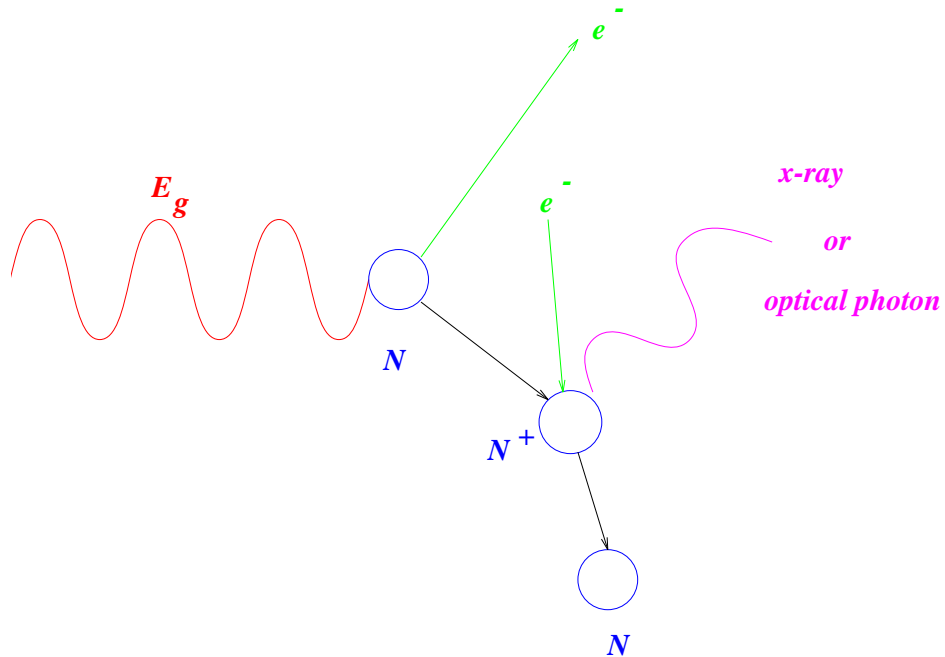


Figure 1: Diagrammatic sketch of a photoelectric process. N denotes the nucleus participating in the interaction.

that atom ionized (see figure 1). Most of the kinetic energy of the photon is transferred onto the electron and can in turn be measured. This allows a straight forward measurement of the photon energy. A discrete photon energy results in a discrete electron energy.

Energy and momentum can not be simultaneously conserved in photoelectric absorption of a photon at a *free* electron. Photoelectric effect thus always occurs on bound electrons, with the nucleus absorbing the recoil momentum. As the nucleus is much heavier than the electron its recoil energy is negligible. Because of the transfer of momentum to the nucleus the cross section of this process has a very strong Z (nuclear charge) dependence of Z^4 - Z^5 . The atom is left in an ionized state that quickly captures an electron with the emission of an x-ray. If the detection medium is dense enough the emitted x-ray is re-absorbed and the full photon energy can be recovered. The γ -ray energy, E_γ , has to be equal or higher than the binding energy, B (sometimes also called the work function), of the electron that is ejected from the atom. The

kinetic energy of the ejected electron, E , is given by:

$$E = E_\gamma - B \quad (1)$$

As the γ -ray energy increases and reaches the binding energy, the photoelectric cross section increases suddenly and as the energy keeps increasing the contribution to the cross section due to this electron decreases leading to so called absorption edges in the cross section energy diagram. For gamma energies smaller than the electron mass $E_\gamma < m_e \cdot c^2$ the total cross section (per atom) is approximately given by:

$$\sigma = 4\sqrt{2}\alpha^4 Z^5 \left(\frac{m_e \cdot c^2}{E_\gamma}\right)^{3.5} \sigma_e. \quad (2)$$

$\alpha = \frac{1}{137.036}$ denotes the fine structure constant, and $\sigma_e = \frac{8\pi r_e^2}{3} = 6.651 \cdot 10^{-25} \text{ cm}^2$ the classical cross section ($r_e = 2.817 \cdot 10^{-15} \text{ m}$ classical electron radius). Read *Leo 2.7.1* for more details.

The important point is that the cross section has a rather strong Z and energy dependence. Although the use of photo effect is the most straight forward way of measuring the energy of photons it can sometimes not be used because e.g. the photon energies are too high and the cross section thus becomes too small.

2.2 Compton Scattering

This is the scattering of a photon off a free electron. In matter electrons are bound. For photon energies much larger than the binding energy (which is typically of order eV) the latter energy can be ignored and the electrons can be considered *free* particles. Figure 2 illustrates this process. The kinetic energy of the out going electron is transferred to the medium and can be measured. A determination of the energy of the incoming photon is only possible if the scattering angle is measured as well. Typically a discrete photon energy results in a continuum of electron energies. The photon energy can thus only be measured indirectly when using Compton scattering.

The energy of the scattered photon E'_γ depends, just like in any elastic scattering, on the scattering angle θ . It is given as:

$$E'_\gamma = \frac{E_\gamma}{1 + \gamma(1 - \cos\theta)}, \quad (3)$$

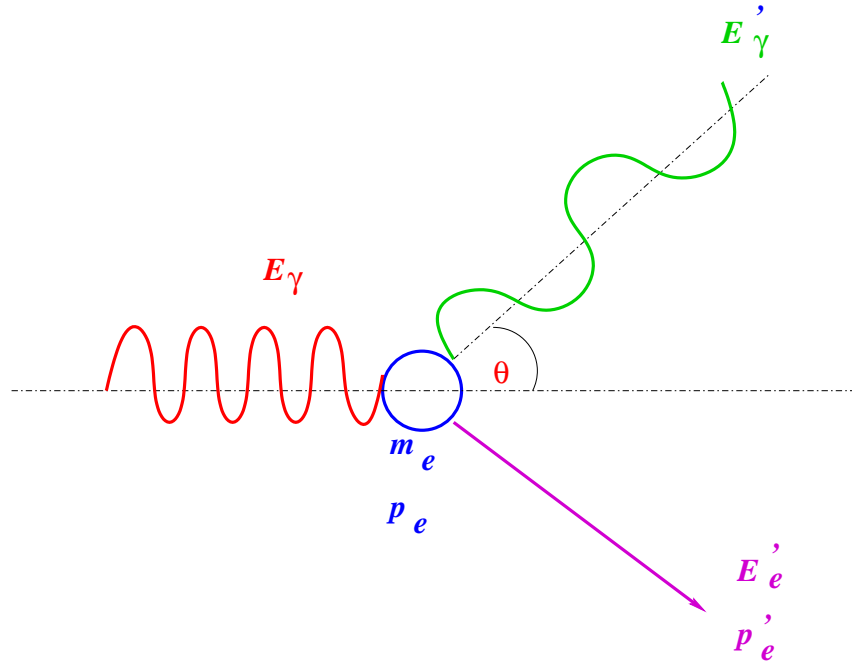


Figure 2: Diagrammatic sketch of a Compton scattering.

with $\gamma = \frac{E_\gamma}{m_e c^2}$.

Problem 1:

Show that the highest energy transfer from the photon to the electron occurs in backscattering ($\theta = 180^\circ$).

The corresponding energy is called the Compton edge.

Problem 2:

Show that the maximal recoil energy, T_{max} , an electron can acquire in Compton scattering is given by:

$$T_{max} = E_\gamma \left(\frac{2\gamma}{1 + 2\gamma} \right) \quad (4)$$

A ^{137}Cs calibration source, used in the lab, emits photons with $E_\gamma = 662 \text{ keV}$. Calculate the energy of the Compton edge.

The differential cross section can be calculated using quantum electrodynamics. It is called *Klein-Nishina* cross section (see *Leo chapter 2.7.2*):

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2} \frac{1}{[1 + \gamma(1 - \cos\theta)]^2} \left(1 + \cos^2\theta + \frac{\gamma^2(1 - \cos\theta)^2}{1 + \gamma(1 - \cos\theta)} \right), \quad (5)$$

$d\omega$ denotes the solid angle element, at the scattering angle θ , into which the photon is scattered. Equation 5 does not depend on the azimuthal angle ϕ . Photons are hence scattered, with equal probability, into a *cone* with opening angle θ . The *width* of the cone is measured by $d\omega$. The total cross section, σ_{Comp} , is obtained by integrating 5 over $d\Omega$ (or over all scattering angles; with the integration over the azimuthal angle ϕ contributing a factor 2π). For electrons bound in matter σ_{Comp} is approximately $\sigma_{Comp} \sim \frac{Z}{E_\gamma^2}$ (per atom). The cross section raises like Z and falls like $1/E_\gamma^2$.

Combining equations 3 and 5 allows to calculate the distribution of the

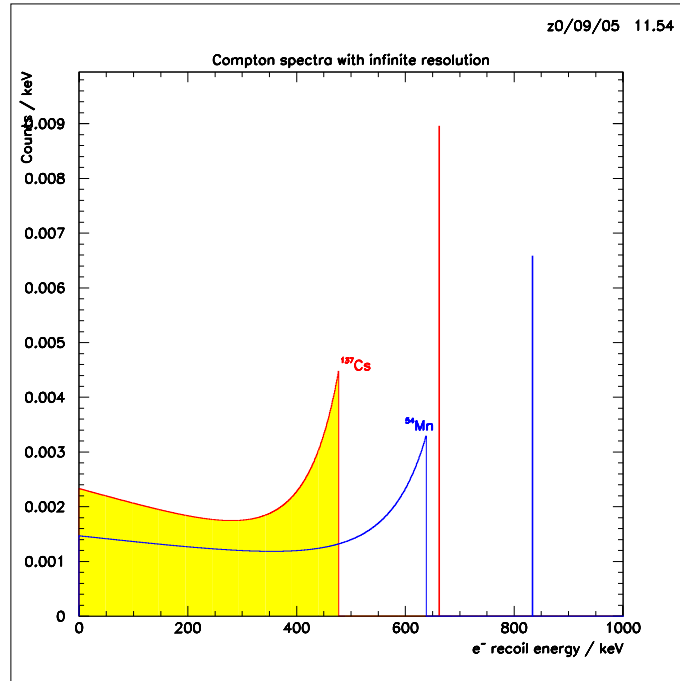


Figure 3: Expected spectral shape for a detector with infinite energy resolution.

electron recoil kinetic energy, T_e , which will be measured in this experiment.

$$\frac{d\sigma}{d\Omega} = \frac{\pi r_e^2}{m_e c^2 \gamma^2} \left[2 + \frac{s^2}{\gamma^2 (1-s)^2} + \frac{s}{1-s} \left(s - \frac{2}{\gamma} \right) \right], \quad (6)$$

with $s = \frac{T_e}{E_\gamma}$. Calculated T_e -distributions are shown in figure 3 for two different gamma-ray energies. The sharp discontinuity of the continuous Compton spectrum corresponds to the Compton edge. That's where the name comes from. The measurement of this edge thus allows us to determine the photon energy *statistically*. We need to collect many Compton scatter events to analyze the statistical distribution of the electron energy. The discrete peaks correspond to total absorption of the gamma ray energy. They have to involve photo electric effect.

2.3 Pair Production

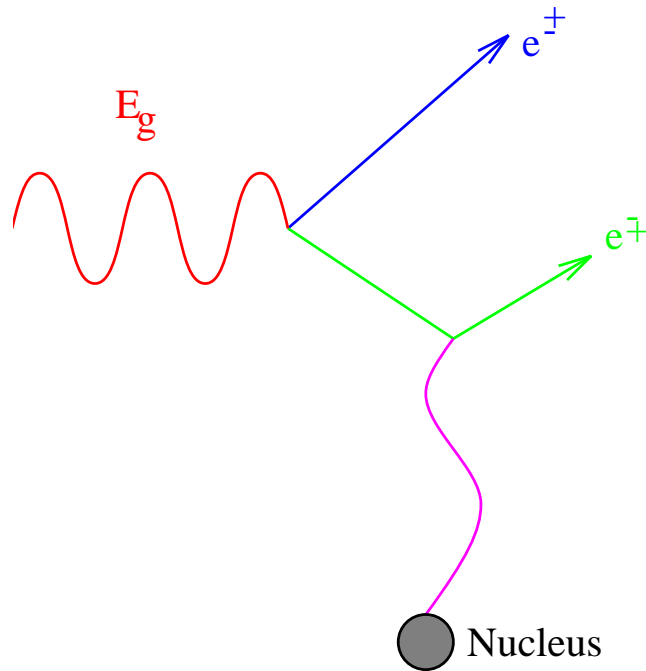


Figure 4: Diagrammatic sketch of Pair production.

This process involves the transformation of a photon into an electron-positron pair, as depicted in figure 4. The charged electron/positron are rapidly losing their kinetic energy to the medium. After being slowed down the positron will form positronium with an electron and eventually annihilate, predominantly into two 511 keV photons. These may or may not in turn be absorbed by the medium. A discrete photon energy hence results in three discrete electron energies superimposed to a continuous distribution: (a) full absorption of the the electron/positron kinetic energies *and* of both annihilation quanta, (b) full absorption of the the electron/positron kinetic energies and partial absorption of the annihilation quanta, and, (c) full absorption of the electron/positron kinetic energies and total loss of one or both of the annihilation quanta. Pair production can thus be used to measure the photon energy.

In order to conserve both energy and momentum there has to be an exchange of energy and momentum with a nucleus or an electron. This reaction requires a minimal photon energy as two massive particles have to be created out of the vacuum. This threshold energy, E_T , is given by:

$$E_T = \frac{(M + 2m_e)^2 - M^2}{2M}c^2 = 2m_e c^2 + \frac{2m_e^2}{M}c^2, \quad (7)$$

where M denotes the mass of the nucleus with which the photon is interacting, m_e denotes the electron/positron mass. It can be seen from equation 7 that for reactions with the nucleus ($M \gg m_e$), $E_T = 2m_e$ as the second term becomes very small. However, pair production can also happen with the participation of an electron. For $M = m_e$ the threshold energy becomes $E_T = 4m_e$. At this higher energy photons can thus interact with nuclei *and* electrons and the pair production cross section increases accordingly. Pair production is only important at relatively high photon energies.

For the calculation of the pair production cross section in matter electron screening of the nucleus is important. The details of this go beyond the scope of this introduction. However, to be complete the total cross section for pair production is given for $m_e c^2 \ll E_\gamma \ll 137m_e c^2 Z^{-1/3}$, if screening is neglected, as:

$$\sigma_{Pair} = 4Z^2 \alpha r_e^2 \left[\frac{7}{9} \left(\ln \frac{2E_\gamma}{m_e c^2} - f(Z) \right) - \frac{109}{54} \right], \quad (8)$$

with $f(Z) \approx a^2[(1 + a^2)^{-1} + 0.20206 - 0.0369a^2 + 0.0083a^4 - 0.002a^6]$ ($a = Z/137$) being a small correction accounting for the Coulomb interaction of

the outgoing Leptons in the electric field of the nucleus. Even for Pb the upper limit for the application of this approximation is with $137m_e c^2 Z^{-1/3} = 16.1 \text{ MeV}$ beyond the energy scale of any photons we are using in the lab. To take into account electron screening requires numerical calculations. The important fact to remember is that this cross section *increases* as Z^2 and also *increases* with the photon energy.

2.4 Total Absorption

Energetic photons interact with matter in all three ways outlined above. The likelihood of any of the interaction types depends on the photon energy and the material used. Photons carry no charge so their interaction with the electrons of a medium is not a continuous (as for charged particles) but a statistical process.

Due to its statistical nature the distance intensity relation $I(x)$ is an exponential for uncharged particles. The so called total mass attenuation coefficient μ_{tot} is given as:

$$\mu_{tot} = \mu_{Photo} + \mu_{Comp} + \mu_{Pair} \quad (9)$$

You can find its energy dependence in figures 5- 9, in units of $[cm^2/g]$. This allows you to estimate the scattering probability using:

$$I(x) = I_0 e^{-\mu_{tot} \rho x} \quad (10)$$

for gamma radiation propagating through matter. ρ , in units of $[g/cm^3]$, is the density of the medium, x (in $[cm]$) the path length. The graphs 5- 9 also tell you the partial cross sections of the different fundamental reaction. Look at the graphs and try to understand the different response of the NaI and plastic scintillators to gamma radiation. A beam of gamma rays can never be completely stopped as possible for charged particles. You can only attenuate it using sufficient absorber.

Problem 3:

What is the the probability that a 662 keV γ -quantum traverses 2 cm of a NaI crystal without interaction?

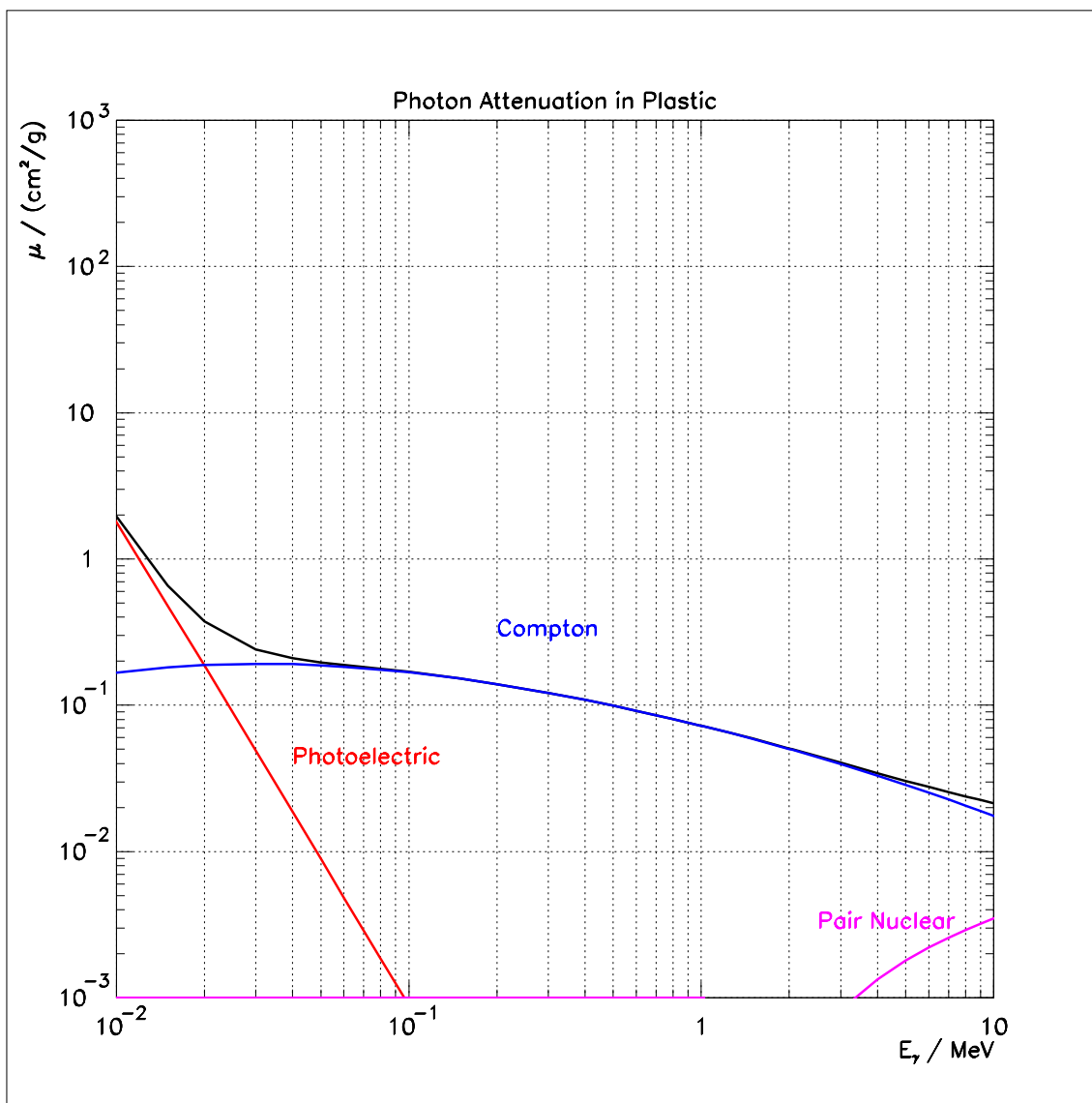


Figure 5: Mass attenuation coefficients for plastic (1.2 g/ml).

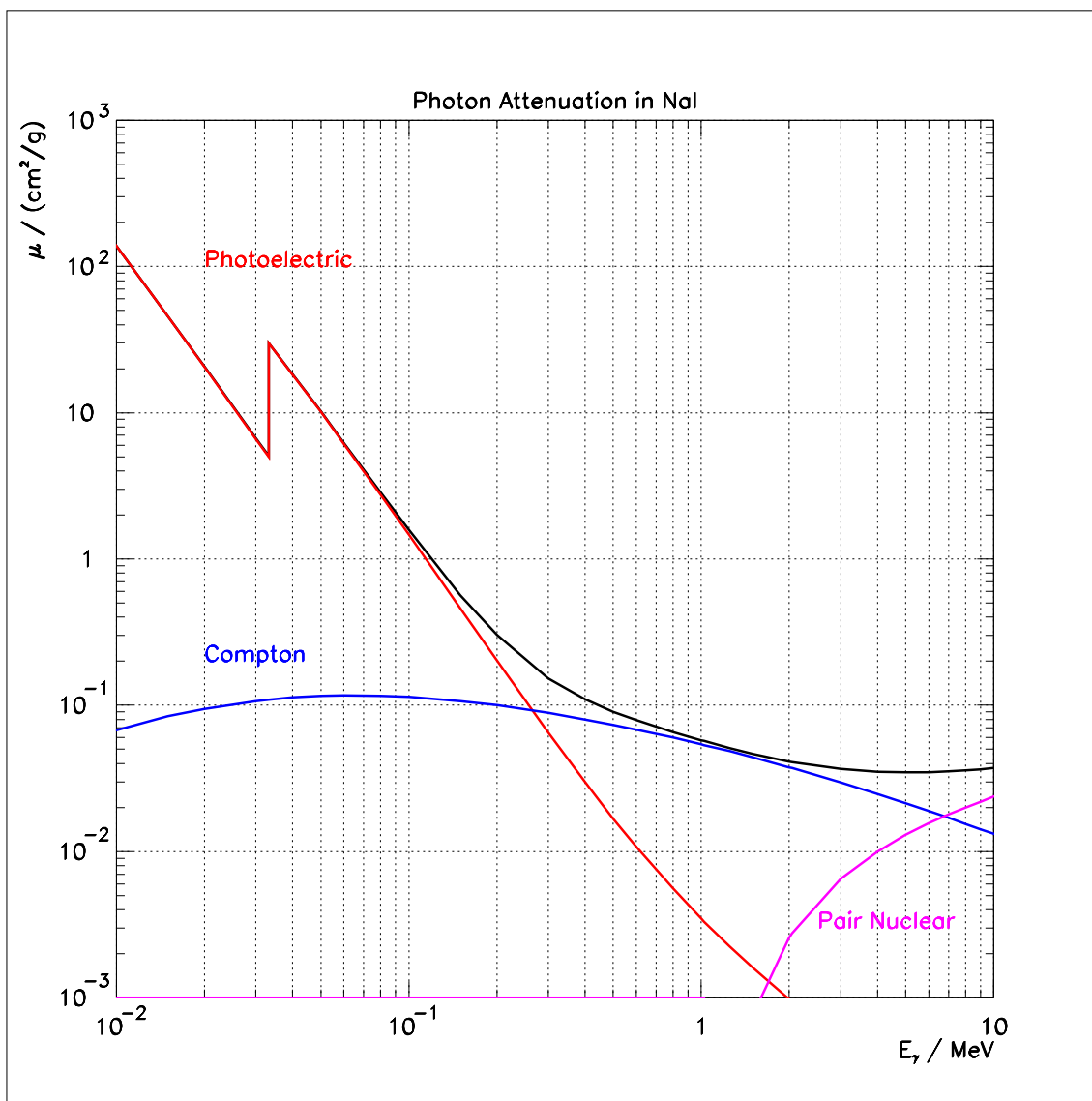


Figure 6: Mass attenuation coefficients for NaI (3.67 g/ml).

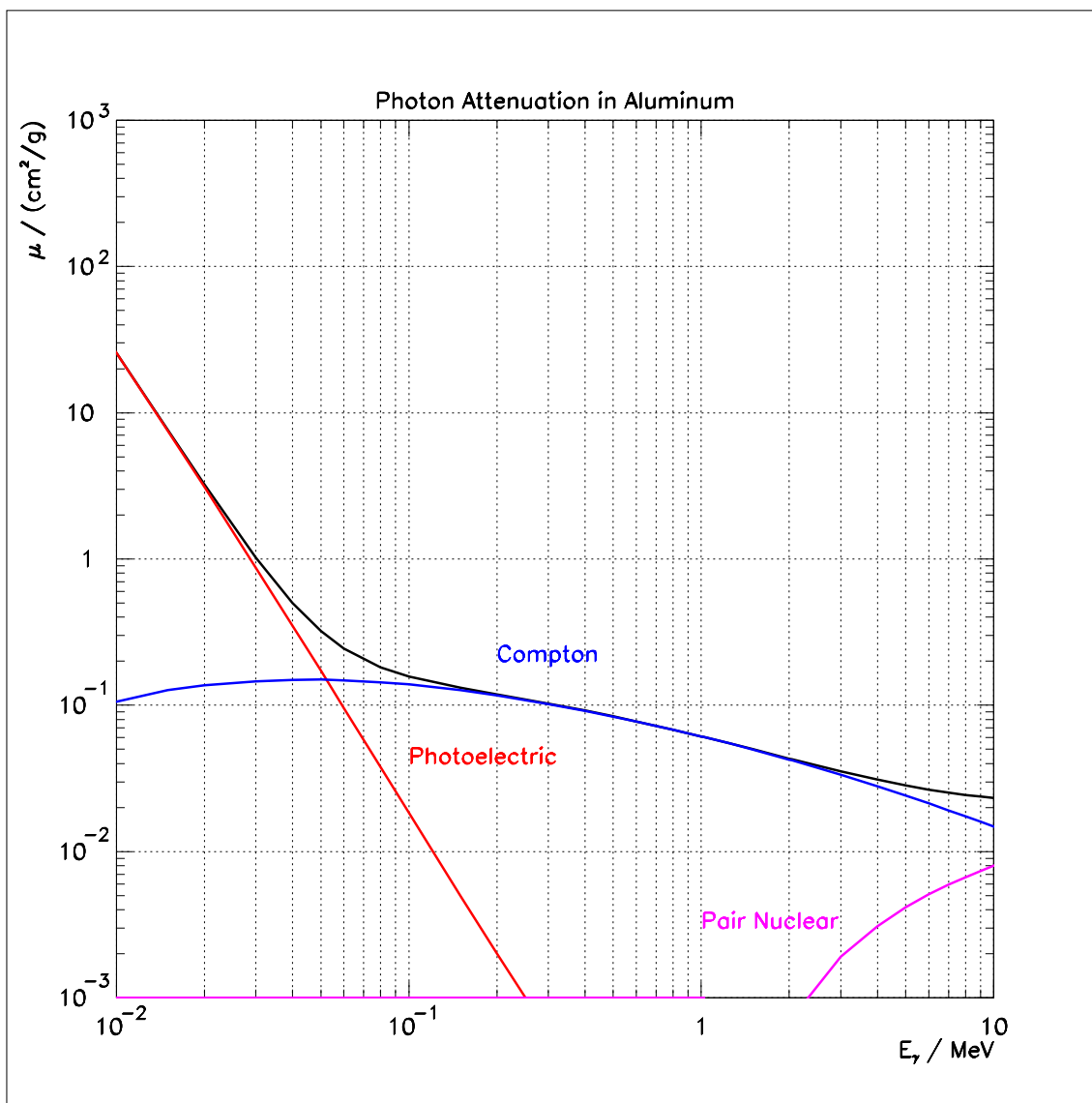


Figure 7: Mass attenuation coefficients for Aluminum (2.70 g/ml).

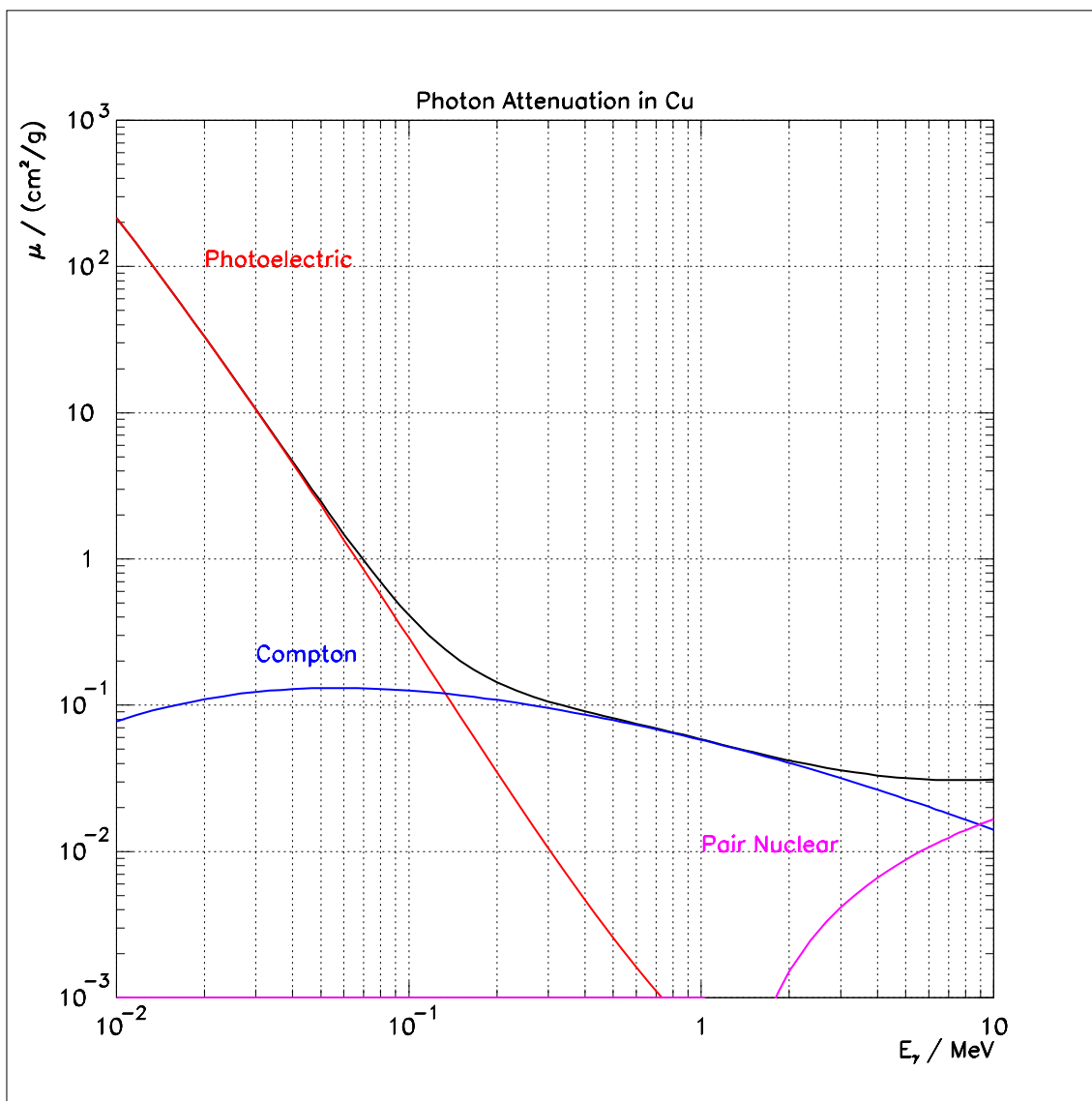


Figure 8: Mass attenuation coefficients for Copper (8.96 g/ml).

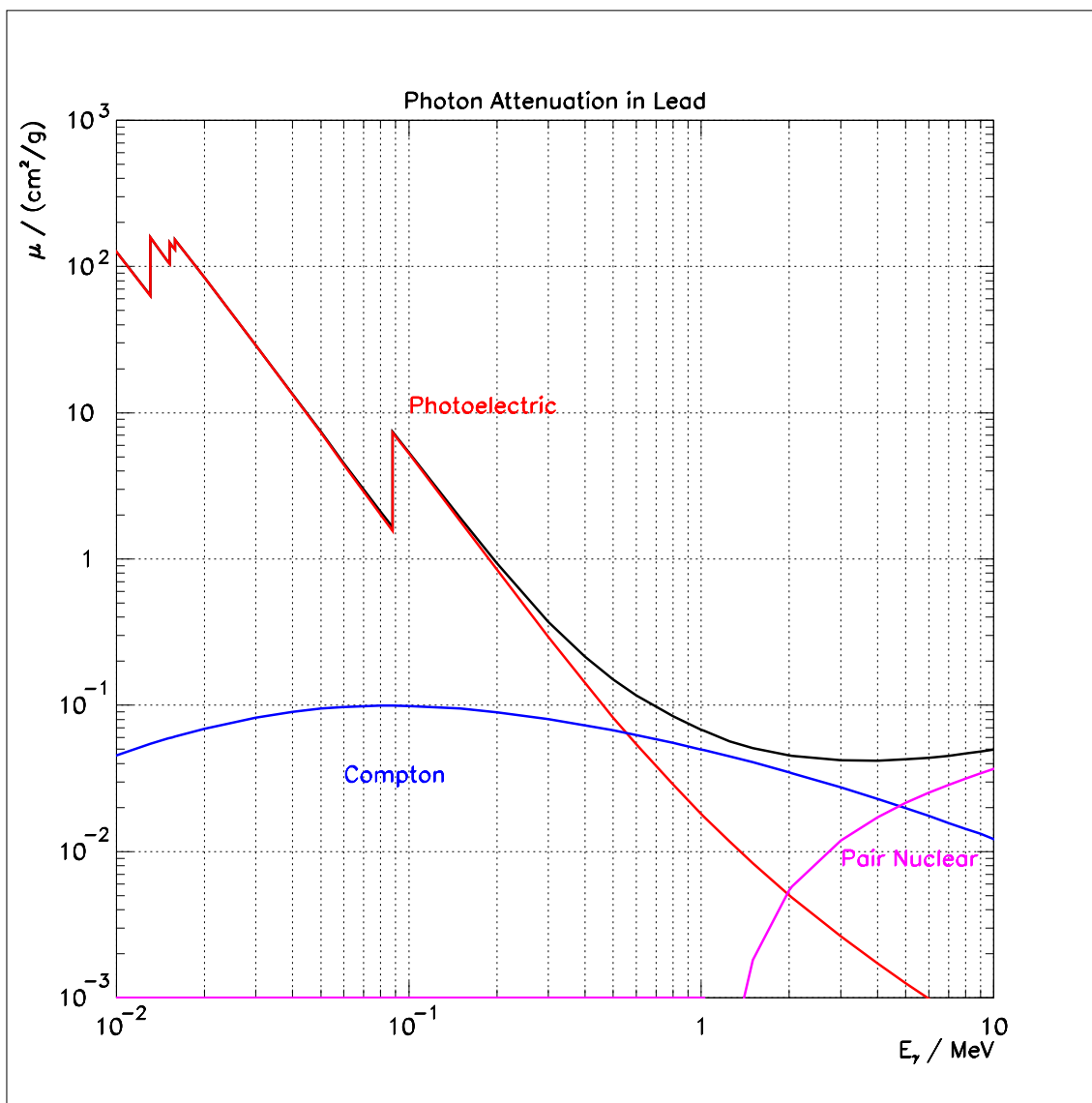


Figure 9: Mass attenuation coefficients for Lead (11.35 g/ml).

3 The used Equipment

In this experiment we are using scintillation detectors to measure the statistical distribution of energies transferred from energetic photons, emitted by various radioactive sources, to the electrons of the detection medium. Fast electronics is used to amplify and shape the resulting short charge pulses which are then in turn histogrammed by a data taking PC. The following sections will give a brief description of all major building blocks of the set up.

3.1 The Observed Spectra

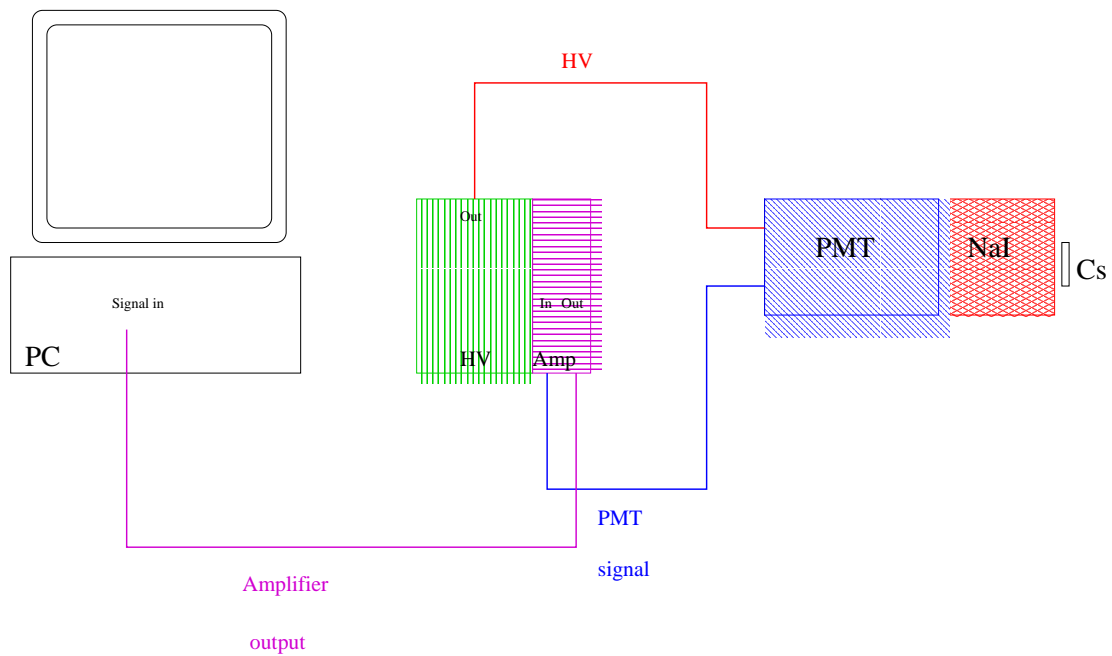


Figure 10: Schematic of the scintillator counting setup.

The source depicted in figure 10 can be any gamma ray emitting source. The scintillation detector converts energy deposited by charged particle into visible light. We have discussed above how these charged particles relate to the original photons emitted by the radioactive sources. Part of this light

is transmitted to the photomultiplier tube (PMT). The PMT converts the visible photons back into electrons by means of photo electric effect. It then amplifies the charge in an avalanche amplification.

You will use two different kinds of scintillators in this experiment:

- **Sodium Iodide NaI** The light output of this detector is an exponential of the form: $light = Ae^{t/\tau}$, where the time constant τ is $0.25 \mu s$.
- **Plastic Scintillator** These scintillators are much faster, all the light is emitted in a very short burst. The current output width is a few nano seconds and in many cases it is determined by the photomultiplier and not the scintillator.

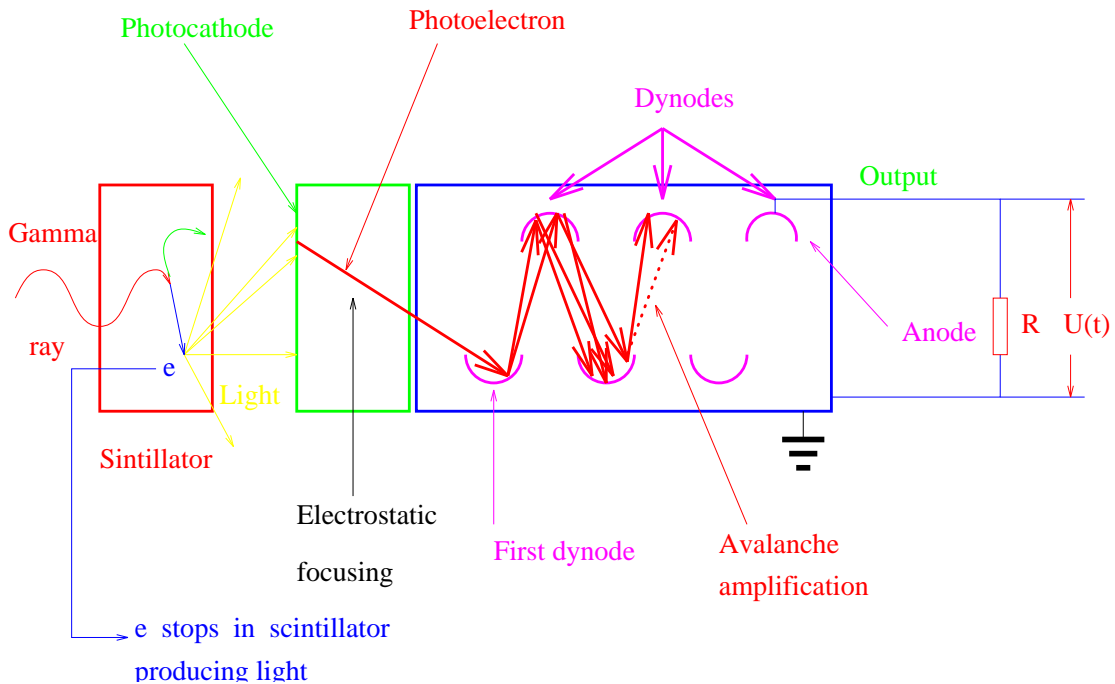


Figure 11: Schematic of the photomultiplier tube.

Figure 11 depicts the photomultiplier tube. The photocathode is a very thin film that converts, through the photoelectric effect, the scintillation photons

into electrons (the efficiency for this is usually 15-30 %). These electrons are focused into a small opening just in front of the first dynode. The dynodes then perform the actual electron multiplication. When an electron with a kinetic energy of about 200 eV hits a dynode, secondary electrons are ejected from it. They are again accelerated and strike the following dynode and so on. The PMT is a charge amplifier. Its gain per dynode g_0 and the number of dynodes n determine its total amplification factor $g = g_0^n$. Depending on the PMT used g can be $10^5 - 10^7$. g_0 depends on the applied voltage. A resistive voltage divider connects the dynodes to the high voltage to give uniform amplification. You can hence adjust the gain with the supply voltage chosen.

The anode collects all electrons and places them at the photomultiplier output. The charge delivered by the PMT is proportional to the energy deposited by the charged particle in the detector. The amplifier (see figure 10) boosts the signal amplitude and shapes the signal. It integrates the input signal and by this converts the *charge* signal into a *voltage* signal. After passing through the amplifier the voltage amplitude is proportional to the energy deposit in the scintillator. Figure 12 depicts a typical voltage signal after the amplifier. Please note the shape of input and output signals of all detectors used into your notebook. Use one of the oscilloscopes available in the lab.

The data taking computer, depicted in figure 10, has an internal *Discriminator*, *Analogue to Digital Converter (ADC)* and *Multi Channel Analyzer (MCA)* to facilitate data acquisition. Whenever the voltage signal exceeds a lower voltage threshold the ADC will start working. This threshold can be adjusted with a small trim potentiometer at the back of the computer.

The ADC provides a digital address proportional to the signal amplitude. It has an input voltage range of 0 to 8 V (that's another reason why we need the amplifier) and provides an integer output number from 0 to 2047 (2^{11} called 11-bit resolution). The MCA then keeps track of how often a certain digital voltage occurred during the run and provides a graphic display which we call the "spectrum". Based on our discussion of the interaction of gamma radiation with matter we would expect spectra depicted in figure 3.

What we actually see is, however, something else. Due to statistical fluctuations the one-to-one relation between signal amplitude and energy deposited in the scintillator is only true on average. The most important source of fluctuations is due to the finite number of electrons emerging from the photocathode. This number is subject to statistical fluctuation. At the photo cathode

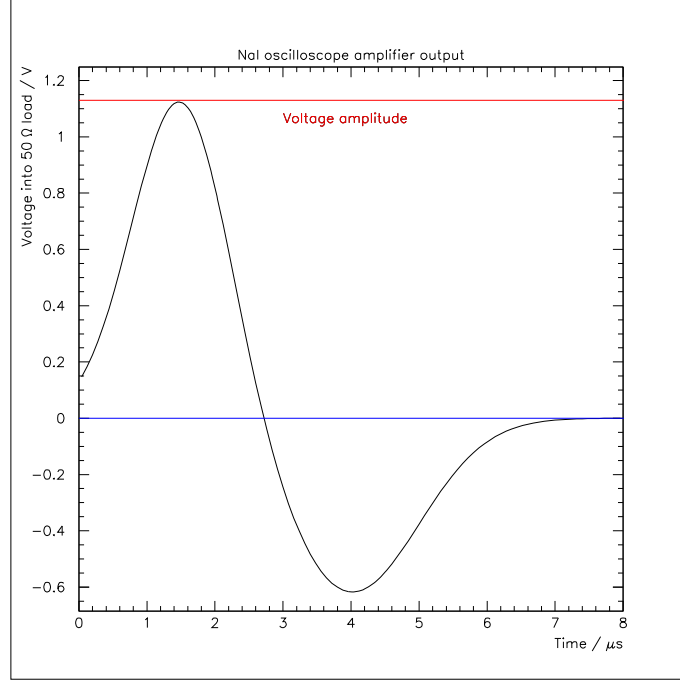


Figure 12: Temporal development of a typical NaI detector voltage pulse as delivered by the amplifier.

the number of information carriers is minimal. A counter-photomultiplier system is characterized by the average energy necessary to produce one photoelectron, E_{PE} (in eV). You will determine this energy for the two detector types available in the lab. The average number of photoelectrons N_{PE} , corresponding to the energy T_e deposited in the counter is given by:

$$N_{PE} = \frac{T_e}{E_{PE}} \quad (11)$$

For signals with less than 20 pe at the photocathode their distribution is given by the Poisson-distribution (see the Appendix for details). Above this value the Poisson-distribution is well approximated by a Gauss-distribution with its width σ satisfying: $\sigma = \sqrt{Mean}$. Using equation 11 we can now express the Gaussian photoelectron distribution $R(T, T_e)$ in terms of energy

T :

$$R(T, T_e) = \frac{1}{\sqrt{2\pi}\sigma(T_e)} \exp\left(-\frac{(T - T_e)^2}{2\sigma^2(T_e)}\right), \quad (12)$$

here T_e stands for the *mean energy* (or the peak maximum) and T for the energy measured for any given event. If we now *assume* that the observed

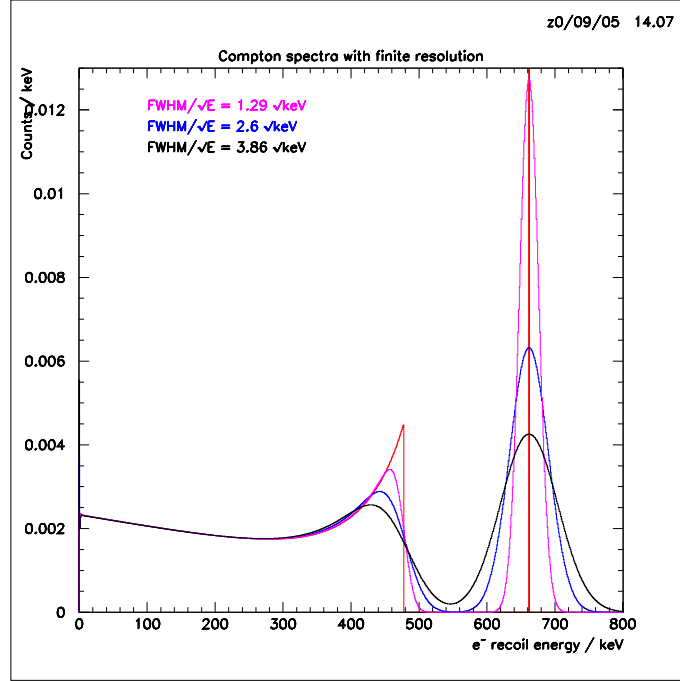


Figure 13: Expected spectral shapes folded with different energy resolutions.

width of the Gauss-distribution is mainly determined by the number of photoelectrons then we can determine the energy required to generate a photoelectron by means of a statistical analysis from the data.

As stated before the width of the Poisson/Gauss-distribution depends on the number of photoelectrons. Let T_e be the centroid and σ_e the width of the charge distribution (associated with a certain full absorption gamma peak) and let G be the total gain (energy calibration) of the detection system. For a Poisson distribution:

$$T_e = G \cdot N_{PE}$$

$$\begin{aligned} \sigma_e &= G \cdot \sqrt{N_{PE}} \\ \Rightarrow N_{PE} &= \left(\frac{T_e}{\sigma_e}\right)^2 \end{aligned} \quad (13)$$

Inserting equation 11 into equation 13 gives:

$$E_{PE} = \frac{\sigma_e^2}{T_e} \quad (14)$$

From this you see that E_{PE} is a direct measure for the energy resolution of a scintillation detector.

We can, to a certain extent, cross check our assumption made on the statistical nature of the resolution. If we measure position and width of several peaks corresponding to different numbers of photoelectrons then a plot of T_e on the x-axis versus N_{PE} (as determined using equation 14) on the y-axis should show a linear dependence. Any non-zero intercept, b , would be interpreted as (the non-statistical) noise contribution:

$$N_{PE} = m \cdot T_e + b, \quad (15)$$

while the slope, m , of the line equals $\frac{1}{E_{PE}}$.

If we determine the widths of various gamma ray peaks and plot them as a function of energy we can determine the energy needed to generate a photoelectron.

Goal of this experiment (besides learning the essentials of a radiation detection) is to determine E_{PE} for both the NaI and the plastic scintillation detector using above approach.

3.2 Data Taking

Before you can acquire data the high voltage (HV) power supply of the NaI or the plastic scintillator, available in the lab, need to be turned on. Before doing this make sure that the HV cable is connected at both ends. Before turning on ask the lab advisor to check! Figure 10 shows you a schematic of the setup.

3.3 NaI Detector

Make sure the NaI scintillator (identified by its Al housing) is connected. After the HV has been turned on, give the photo tube of the detector a few minutes to warm up and stabilize. Put a ^{137}Cs calibration source in front of the detector you are going to use. Make sure that the detector to which you have applied HV is connected to an amplifier input and that in turn the output of that amplifier is connected to the input connector of the Tennelec PCA board located in two of the PC's. Use only the proper coaxial transmission lines equipped with BNC connectors. These have a bayonet like locking mechanism (close by twisting clockwise).

After you have verified proper wiring remove the input cable from the amplifier and connect it to one of the oscilloscopes. Get its adjustments right and inspect the timing and amplitude of the signal delivered from the photo multiplier tube (PMT). Sketch the signal shape together with its amplitude and 10-90% fall and rise times into your notebook. This figure, along with a similar one take with the plastic scintillator will be part of your lab report. After you have done that reconnect the cable to the amplifier.

The functions of the Multichannel Analyzer are described in the manual located next to the data taking PC. Read the sections describing its operation and go through the tutorial. After having done this you are ready for serious data taking.

Here a short compilation of the basic operations:

- “F1” turns the data acquisition on and off.
- “Ctrl F2” deletes a previously accumulated spectrum.
- “F3” allows you to preselect a measuring time in seconds.
- “Alt F” activates the file menu. Select “Save ASCII” and chose an appropriate file name (always use file extension “.asc”).

Activate the read out software by double clicking on the PCA shortcut on the PC's desktop.

4 Tasks for the first day

4.1 Get started

It is a good idea to familiarize yourself with the equipment before you start with quantitative measurements. Take spectra with several sources try to understand what you see. Play with the various options of the data taking system.

4.2 Counting Statistics

The goal of this part of the experiment is to demonstrate that results of a counting experiment are subject to statistical fluctuations. These are *not* due to any imperfections in the data acquisition but instead a natural uncertainty inherent to any counting application. The mean and standard deviation (σ) of the distribution stand in a well defined relation to each other: $Mean = \sigma^2$. Goal of this measurement is to demonstrate this relation. This is instructive as the energy measurement with the scintillation detector is nothing else but counting electrons, as explained earlier in detail. We will be using the same relation between mean and width later for our determination of E_{PE} . So this counting experiment will demonstrate the validity of the approach.

Using the ^{137}Cs source collect a spectrum for 10 and 100 s. Do you note a difference in the appearance of the data when comparing a data set taken during 10 or 100 s of data taking? Explain this difference. When repeating our counting experiment we don't get the same number of counts all the time! This is caused by the fact that nuclear decays are of a statistical nature. We can determine the number of decays only on average (see the Appendix on Counting Statistics).

We want to quantify the fluctuations present in the counting data. To do so we repeat a 1 s data taking with the Cs 50 times. It is very important to not change anything (e.g. the location of the source or measuring time) which could influence the number of counts taken. Hold the ^{137}Cs source in a lab clamp. Adjust 10 mm distance to the Al cap of the NaI detector. After concluding each one second run note the total number of events recored in your lab book. Type the number of events into the first column of a data file

as you take the data.

The total number of events can be obtained by defining a so-called “region of interest” (ROI). To define one or many ROI’s move the cursor (move the mouse) to the left end of the spectrum (or wherever you want to start your ROI) and press “F9”. Then move the cursor to the right end of the spectrum and press “F10”. You will note that PCA high lights the chosen range in red. Note the gross number of counts in your notebook as you go. For the following 49 runs leave the ROI unchanged. To delete a ROI just put the cursor between its limits and press “Del”.

Alternatively you may use the scaler to determine the number of events per each run.

After finishing your 50 ^{137}Cs runs take another 50 (1 s data taking time) runs with 50 mm, 100 mm, 150 mm and 200 mm distance to the detector. For each series of runs note the number of events in your log book. For each distance type the event numbers into a new column of your data file.

Finally take 50 one second runs without any source near the detector. Leave your ROI’s unchanged. Again note the gross number of counts for every run in your note book and the data file as you go along.

For every of your 6 counting sequences plot a frequency distribution using PAW on the work station (see description below) and determine the mean and standard deviation of each distribution. After you have done that plot the derived variances against the mean. Is there a functional relation between the two quantities? If so what?

5 Tasks for the second day

5.1 Energy Calibration

NaI Detector

Collect spectra with all the sources available in the lab. Select at least 100 s data taking time. Tape the sources to the center of the NaI housing. Analyze the data and derive an energy calibration of our setup by performing a linear fit of the positions of the peaks (apparent in the data) and tabulated γ -energies which you will look up on the web.

When you are done with a particular source save the collected spectrum as

an ASCII file as described before. Note the file names, measuring time in your notebook together with the activity of the source used which you can find on the PH491 web site. Make sure that all sources are taped to the same location at the center of the cylindrical detector face.

There is one unknown (un-labeled) source in the lab. Measure the γ -spectrum of this source and try to identify it.

5.2 Energy resolution

Above we have discussed how a measurement of the energy resolution of the detector allow you to quantify the number of electrons created in the PMT. Apply the discussed technique and determine the energy required to create a photo electron from the energy calibration data set using equation 14.

6 Tasks for the third day

6.1 Energy Calibration

Plastic Scintillator

Connect the input of the amplifier to the plastic scintillator and its output with the input of the data taking PC. Make sketches of the pulse shape as determined with the oscilloscope of the PMT output and the amplifier output. Do you note the difference to the NaI?

Put the ^{137}Cs , ^{54}Mn ^{60}Co and ^{24}Na sources on top of the plastic scintillator and collect spectra. You note that there are no full absorption peaks visible. Give an explanation why this is so. The only way to determine the energy calibration and E_{pe} is to analyze the Compton edges. As you can see in figure 14 there is no prominent feature which can be associated with the Compton edge (plotted in red) once we have folded the original distribution with some energy resolution. However, all folded distributions have one point in common which is close to the Compton edge E_{Comp} . If we fit the falling edge of the plotted distributions with a Gaussian this point corresponds to $E_{Comp} \approx Position_{fit} + \sigma_{fit}$. This gives us a recipe to analyze the data. Moreover the energy resolution of the detector $\sigma(E_{Comp})$ is given

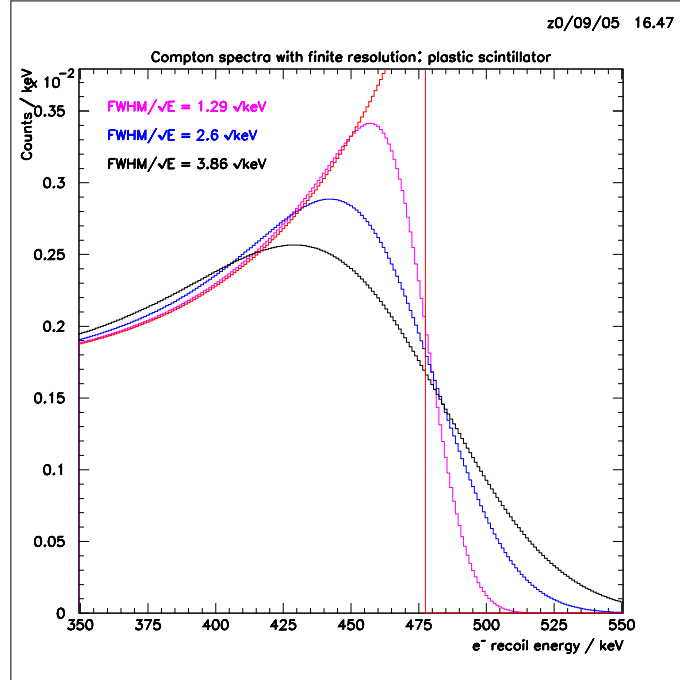


Figure 14: Expected Compton spectra as recorded with the plastic scintillator folded with various energy resolutions.

as $\sigma(E_{Comp}) \approx 0.9 \cdot \sigma_{fit}$.

Plot the fitted $Position_{fit} + \sigma_{fit}$ versus the energy of the Compton edge which you will need to calculate using equation 4. In what relation do these quantities stand to each other (linear, quadratic...)? Derive the energy calibration from this plot.

6.2 Energy resolution

Plot the measured $\sigma(E_{Comp})^2 / (Position_{fit} + \sigma_{fit})$ as a function of E_{Comp} and determine E_{PE} as discussed for the NaI detector.

7 Data Transfer via the Computer Network

Transfer your spectra from the data taking computer to the UNIX workstation using FTP. Click on the FTP short cut on the desk top. The PH491 account will come up automatically. You will have to enter the password and then click "OK". Once you are logged into the UNIX machine click on the spectrum (PC side) you want to transfer. Then click on the arrow pointing into the direction of the workstation. That's all! You have now saved the data on disk. Transfer all your data and then exit FTP by clicking on "Close" to close the connection. End the FTP session by clicking on "Exit". In the next step you have to open a UNIX graphics terminal. Click on the X-Win32 short-cut. A login window will come up. Enter the password of our group account. The terminal emulation should now open a terminal window. If it also opens a dialog window close it. It is, at this point, advantageous to open another window on the UNIX workstation. To do so type "xterm &" at the UNIX prompt. The &-sign will run this as a background job so your UNIX window will still be available for input. In UNIX "ls" will show you a file list, "pwd" will tell you where you are in the directory structure and "cd" will allow you to change the directory.

One manipulation of the data file will be necessary to distinguish file header from data (PAW will choke upon reading the data if you fail to go through this). Load the data file into the file editor EMACS. E.g. if you want to process a previously created spectrum "na22.asc" type: "emacs na22.asc" The editor will now show you the content of the data file in a separate window. Add "!" in the first column of the first n lines containing comments generated by the PCA program. Data lines contain 3 columns of integer numbers separated by commas. To save your changes click on "Files" and then select "Save Buffer". Exit EMACS by clicking on "Files" select "Exit EMACS". You may read and manipulate multiple files in one EMACS session.

8 Plotting and Data Analysis with PAW

You are now ready to plot and analyze your data using PAW. Start your PAW session by typing "pawX11" at the UNIX prompt. PAW will ask you for the workstation type. Just hit "return". This should start a new graphics window labeled "HIGZ". Go with the mouse to its upper left corner and

reduce its size by clicking and holding the left mouse button. If the HIGZ window doesn't pop up you are in trouble and have to call the lab supervisor for help.

A tutorial in how to use PAW can be found on the PH491 web page. To get you started to simple MACROs called "plot-data" and "plot-histogram" have been prepared to serve as an interactive interface between you and PAW.

To plot our file "na22.asc" enter "execute plot-histogram" at the PAW

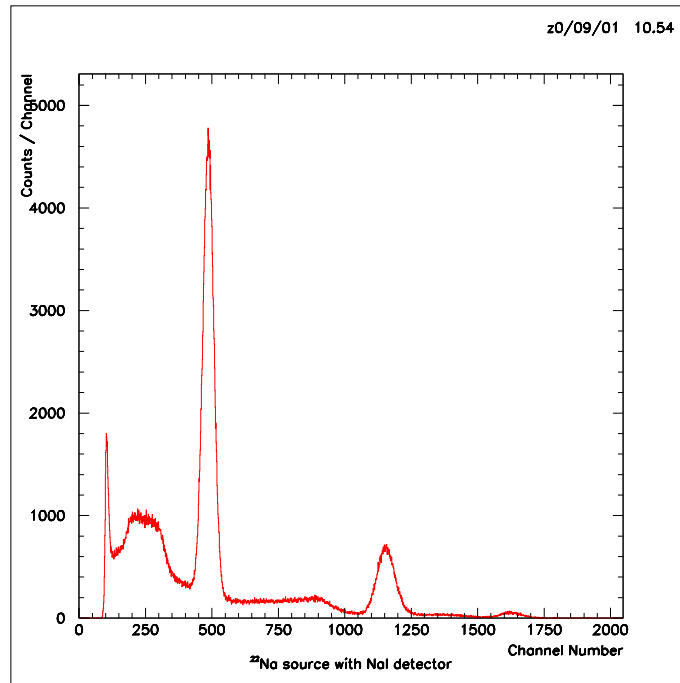


Figure 15: ^{22}Na spectrum recorded with the NaI scintillator as displayed by PAW. The MACRO "plot-histogram" was used.

prompt. The MACRO (called kumac for reasons only known to the folks at CERN) will ask you how many histograms you want to read. Give e.g. 1. You can read more histograms in one session if you want. Histograms are identified by numbers. Our MACRO starts numbering at 100 and increments by 1 for each further spectrum. You will now have to enter the name of the file you wish to read, na22.asc in this example. Now add a short comment which identifies e.g. what detector and source has been used. This comment will be shown with the histogram when you plot it. PAW will now

plot the data. Figure 15 shows how this will look like. Upon start-up “plot-histogram” will show a VERY brief comment. Should you have missed it you can re-display the part of the display buffer by typing “Shift Page Up” until you are there.

For further data analysis it is useful to have an expanded view of the peaks

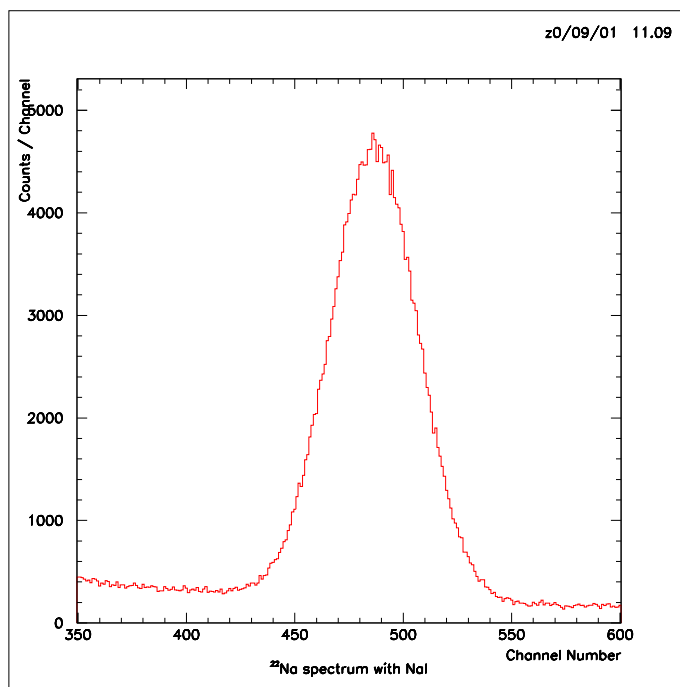


Figure 16: ^{22}Na spectrum expanded around the lower of the two peaks.

which interest us for our energy calibration. Plot and expanded view by “hi/plot 100(350.:600.)” e.g. for our ^{22}Na spectrum (this will zoom into the lower of the two peaks). To get the axis labeled type “atitle 'Channel Number' 'Counts / Channel' ”. Figure 16 shows such expanded plot. The final step is now to fit the data with a Gauss function plus linear background in order to quantify its parameters: amplitude, position and width. In a fit we try to find the best possible parameterization of the data through a pre-defined function. We have to either guess this function or better determine from basic principles what that function should be. A commonly used technique is the so-called χ^2 -minimization. Appendix A gives a brief description of this technique. You may skip over it if you want. For in-depth study read the Primer.

8.1 Data Analysis in PAW

You may now perform a χ^2 -fit to the data. Determine the parameters of the Gaussian which best fits the data through: “hi/fit 100(450.:520.) g”. If you now issue the command “hi/plot 100(350.:600.)” you get the plot displayed in figure 17. We determine a peak integral of $(2.45 \pm 0.03) \cdot 10^5$ counts from

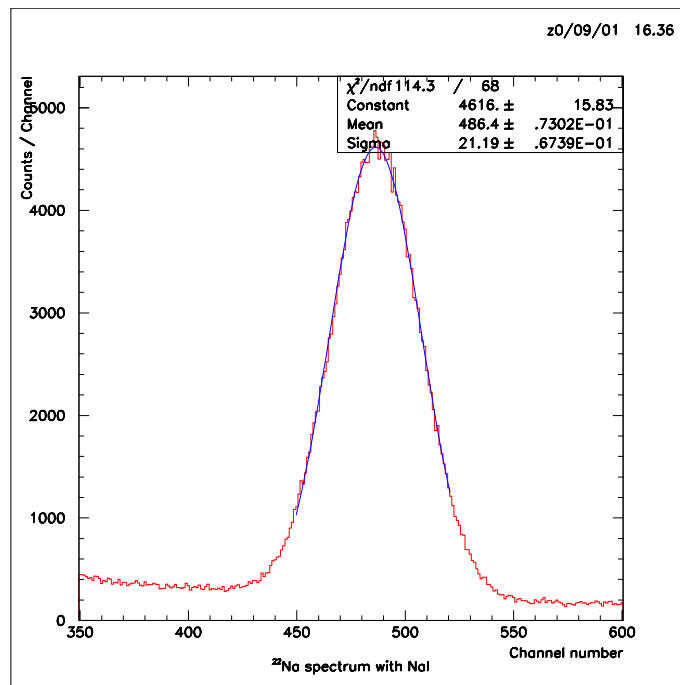


Figure 17: ^{22}Na spectrum expanded around the lower of the two peaks with preliminary fit.

this particular fit. The fit is quite OK but you note that the reduced χ^2 is 1.68 indicating a problem. This is due to the fact that the data is not well described by a Gaussian. At the low and high energy wings of the peak we clearly see a linear background onto which the peak is superimposed. We can improve our description of the data by using a more complex fit function. To do so we need to give reasonable starting values to the fitting

routine. Do that by creating a parameter vector containing those start values: “vect/create par5(5) r 4616. 486. 21.”. You note that these are the parameters we obtained in our preliminary fit. We now perform a new fit: “hi/fit 100(400.:580.) g+p1 ! 5 par5”. This fits a Gaussian plus a first degree polynomial to the data. Figure 18 shows the result. You note that we didn’t

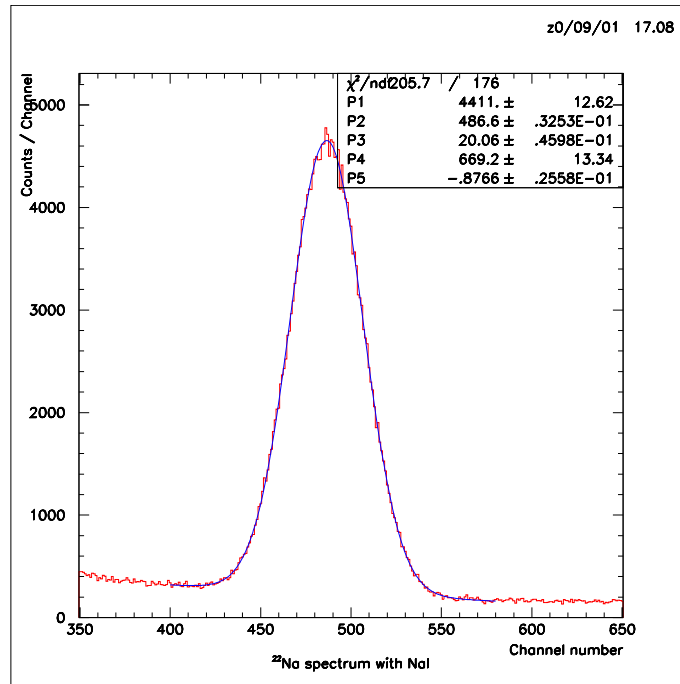


Figure 18: ^{22}Na spectrum expanded around the lower of the two peaks with final fit.

put starting values for the “p1” term into the 5-dimensional vector “par5”. That works in most cases. If not perform a “p1” fit near the peak and enter the result of this fit into vector par5 as shown above (careful the ordering matters). Our new peak integral is now: $(2.22 \pm 0.08) \cdot 10^5$, a 10% difference compared to the preliminary fit. For peaks superimposed to a larger background this difference can be even larger. Now analyze all data files you collected.

8.2 Analysis Tasks

8.2.1 NaI Data

For the repeated source runs calculate the mean and standard deviation for every series of runs. Use the “plot-data” MACRO to plot the data distribution. Now plot the square of the standard deviations as a function of the average number of events collected per each run. Do you obtain a linear relation? If so you have just demonstrated that the fluctuation observed in counting follow the Poisson-distribution.

Fit all gamma ray peaks in your spectra. Plot the gamma energy versus the fitted peak position. Fit the data to polynomials of 1, 2, 3 and 4th degree. Do you observe any improvement in the fit quality. Plot the peak widths, as determined from you fits, versus the fitted peak positions. Plot the squares of the widths as a function of the the fitted peak positions. Do you obtain a linear relation with non-zero intercept? Determine the energy needed to create a photoelectron from this fit.

8.2.2 Plastic Scintillator Data

Determine the energy calibration of the plastic scintillator. For this you have to first calculate the energy of the Compton edges for the various source used. Then plot the Compton edge position (determined in the way discussed before) versus Compton edge energy. Fit the data with polynomials of 1, 2, 3 and 4th degree. Determine E_{PE} as for the NaI detector. Is E_{PE} different for the two detector types?

9 Appendix A: χ^2 Fitting

Let ch_i be the channel number (x-variable), y_i the number of counts found in this channel (y-variable) and $f(ch_i, y_i, p_j)$ a function describing the data. Here the values p_j are the parameters of the function f . The problem is to find those p_j which best describe the data. Once you have found reasonable p_j -values you are able to summarize the information contained in the data with just a few numbers and their associated uncertainties.

A good description of the data is obtained when $[y_i - f(ch_i, y_i, p_j)]^2$ or the difference between the data and our chosen parameterization is minimal. We

introduce the square to avoid that positive and negative difference cancel each other. The fact that χ^2 is positive definite ensures that it measures the true sum of the deviation of the parameterization from the data. To obtain the best possible fit we want to attach weights to each channel. Those having the smallest uncertainty σ_i^2 should have the largest weight. For reasons which will become transparent once you have evaluated the scatter of your repeated source runs around their mean we chose $\sigma_i^2 = y_i$. χ^2 , called the test statistics, is given by:

$$\chi^2 = \sum_i \frac{[y_i - f(ch_i, y_i, p_j)]^2}{y_i} \quad (16)$$

The range of i denotes those channels you wish to include in your fit, while j includes all parameters of the chosen fit function. The best fit, or minimal χ^2 , in the space of all possible functional parameters p_j is obtained by solving the system of equations defined by:

$$\frac{\partial \chi(p_j)^2}{\partial p_j} = 0. \quad (17)$$

for all j . PAW will perform this numerical minimization for you by calling the minimization package MINUIT.

Our fit curve $f(ch_i)$, chosen is a Gauss curve superimposed to linear background, is hence defined as:

$$f(ch_i) = (p_1 \cdot \exp[-\frac{(ch_i - p_2)^2}{2 \cdot p_3^2}]) + p_4 + p_5 \cdot ch_i \quad (18)$$

In this parameterization p_1 is the amplitude of the peak, p_2 its position and p_3 its width (σ). The total peak area is A is given as: $A = \sqrt{2\pi} \cdot p_1 \cdot p_3$. The full width at half maximum (FWHM) is given as $FWHM = 2\sigma\sqrt{2\ln 2}$.

The parameters of the Gauss function have a physical meaning: the peak area measures the overall strength of the peak (number of counts), the position the energy of the γ -line and the width characterizes the energy resolution of the detector. You should avoid fits where the physics meaning of the fit function are not well defined. These are merely parameterizations.

The absolute value of χ^2 measures the “goodness” of the fit. In a good data description $\chi^2/(N-M)$ (N is the number of points used in the fit, M the number of unknown parameters determined from the data), often call the “reduced” χ^2 should be around 1. Values much larger than 1 indicate a poor

choice of the fit function, values much smaller than 1 an underestimate of the fluctuations present in the data. Both are warning signs.