Test of the KamLAND Energy Calibration using ^{214}Bi

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Abstract

We investigate the energy distribution resulting from ^{214}Bi decays distributed throughout the KamLAND liquid scintillator to provide an off-axis test of the energy calibration. Bi-Po delayed coincidences are used to isolate the ^{214}Bi decays. In this note we show that the mean energy of the Bi decays is robust against variations in threshold and energy resolution, and out to a radius of about 5 m offers a reasonable signal to background ratio.
Our study indicates that the measured mean energy exceeds the calculated mean total energy release in ^{214}Bi β-decays by 3.0%. The uniformity of the energy scale is further found to be on the order of $\pm 1.6\%$ for $0 \le R \le 5.0\, m$.

1 Motivation

The lack of off-axis energy calibration data does not allow to test the KamLAND energy scale throughout the entire detector volume. To provide such test we examine the energy distribution of ^{214}Bi β-decays. These are convenient as the Bi is distributed over the entire detector. Tagging the correlated ^{214}Bi-^{214}Po sequence allows to isolate these decays and arrive at a relatively background free data set. The analysis of the time distribution between

prompt Bi and delayed Po decay allows to estimate the random background. Testing the energy of neutron capture gammas throughout the detector suffers from the fact that the events of interest typically have been preceded by a very large energy deposit. This in turn might leave the detector in an atypical state. The Bi analysis, however, is performed with a "quiet" detector, just as in the case of neutrino events.
The examination of the energy distribution of Bi decays therefore allows to test the uniformity of the energy scale throughout the entire detector. In addition comparison to an expectation value provides another input into the definition of the absolute energy scale. At the time this study was performed a comprehensive Monte Carlo model was not available. However, the large size of KamLAND makes it an almost ideal calorimeter for all particles emitted in Bi decays. This is not true for vertices near the balloon. For this reason this test could not be extended beyond a spherical radius of 5 m. The total energy release is modeled using the KamLAND Monte Carlo event generator. It is assumed that all energy is absorbed by the detector. Our test of the energy scale hence includes no corrections for light quenching (and it needs no correction for source absorption and shadowing). This assumption is equivalent to comparing the measured Zn and Co peak positions to the tabulated gamma energies.

2 Energy Model

Figure 1 depicts the distribution of particle kinetic energies as modeled by the KamLAND ^{214}Bi Monte Carlo event generator. Shown in red is the energy distribution of the betas, in blue that of the gammas. The purple spectrum shows the distribution of the total energy. We note the broad peak at 2.1 MeV with only a minor tail extending to lower energies. The mean energy, $\langle E\rangle$ determined for this distribution should therefore be rather insensitive to threshold cuts (as long as the threshold is situated in the low energy tail) and, due the broadness of the peak, to the energy resolution. We evaluate the mean energy in the standard way as:

$$\langle E\rangle \;=\; \frac{\sum_i E_i \cdot N_i}{\sum_i N_i} = \frac{\sum_i E_i \cdot N_i}{N}, \tag{1}$$

where E_i denotes the energy associated with the i-th bin and N_i the number of counts found in the same bin.

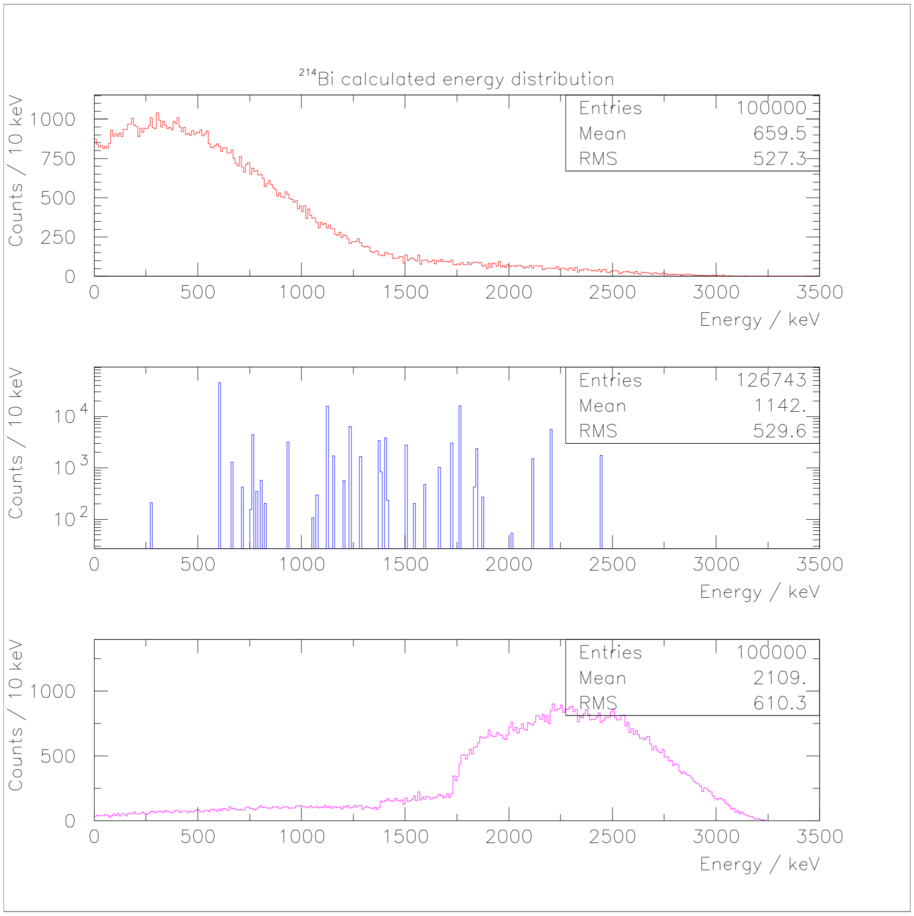

Figure 1: Calculated energy release in ^{214}Bi beta decays. In blue (top): the distribution of betas, in red (middle) the distribution of the gammas. The lower panel (purple) shows the distribution of the total energy.

We use the the mean energy determined for the summed energy release to compare to measured energy distributions in Bi-Po delayed coincidences.
In order to compare measured ($\langle E_{Exp} \rangle$) and calculated ($\langle E_{Mod} \rangle$) mean energies we need an estimator for the statistical fluctuation to be expected for $\langle E_{Exp} \rangle$. The statistical uncertainty in $\langle E_{Mod} \rangle$ can be made arbitrarily small by generating more events and is therefore negligible. The statistical fluctuation of the mean energy, $\sigma_{\langle E \rangle}$, can be estimated by making the following assumptions:

1. There are enough events (N_i) in each bin i so that the expected fluctuations in the number of counts, s_i, governed by the Poisson distribution, can be approximated by a Gauss distribution with $s_i^2 = N_i$.

2. The averaging function is well behaved, meaning that its derivative doesn't fluctuate too much over s_i.

Under these assumptions $\sigma_{\langle E \rangle}$ is given as:

$$
\begin{aligned}
\sigma^2_{\langle E \rangle} &= \sum_i s_i^2 \cdot \left(\frac{\partial \langle E \rangle}{\partial N_i} \right)^2 = \sum_i N_i \cdot \left(\frac{\partial \langle E \rangle}{\partial N_i} \right)^2 \\
&= \sum_i N_i \cdot \left(\frac{E_i \cdot \sum_j N_j - \sum_j E_j \cdot N_j}{(\sum_j N_j)^2} \right)^2 \\
&= \sum_i N_i \cdot \left(\frac{E_i \cdot N - \sum_j E_j \cdot N_j}{N^2} \right)^2 \\
&= \frac{1}{N^2} \cdot \sum_i N_i \cdot (E_i - \langle E \rangle)^2 \; = \; \frac{(RMS)^2}{N},
\end{aligned}
\tag{2}
$$

where RMS denotes the variance of the energy distribution as displayed in the figures.
In order to verify this simple consideration we performed a Monte Carlo study of the mean energy of the Bi distribution. 100, 1000 and 10000 Bi decays were randomly created and the mean energy, the variance of the energy and the estimated uncertainty of the mean were determined. This determination was then repeated 100 times to compare the true fluctuation of the mean with the average of our estimate of the fluctuation. Figure 2 summarize this repeated analysis of Monte Carlo data. We see that the variance found for the repeated analysis of the mean energy is indeed well described by our expression given in equation 2.

The comparison of measured and calculated energy distributions for Bi is not a full Monte Carlo but an approximation based on the assumptions outlined above. For the determination of the mean energy the calculated spectrum is folded with a Gaussian energy resolution. For the analysis we used: $\sigma(E)/\sqrt{E} = 0.073$, as per the Japanese determination of the energy resolution. To see how important these factors are for the the Bi analysis we calculated the mean energy for varying thresholds and different energy

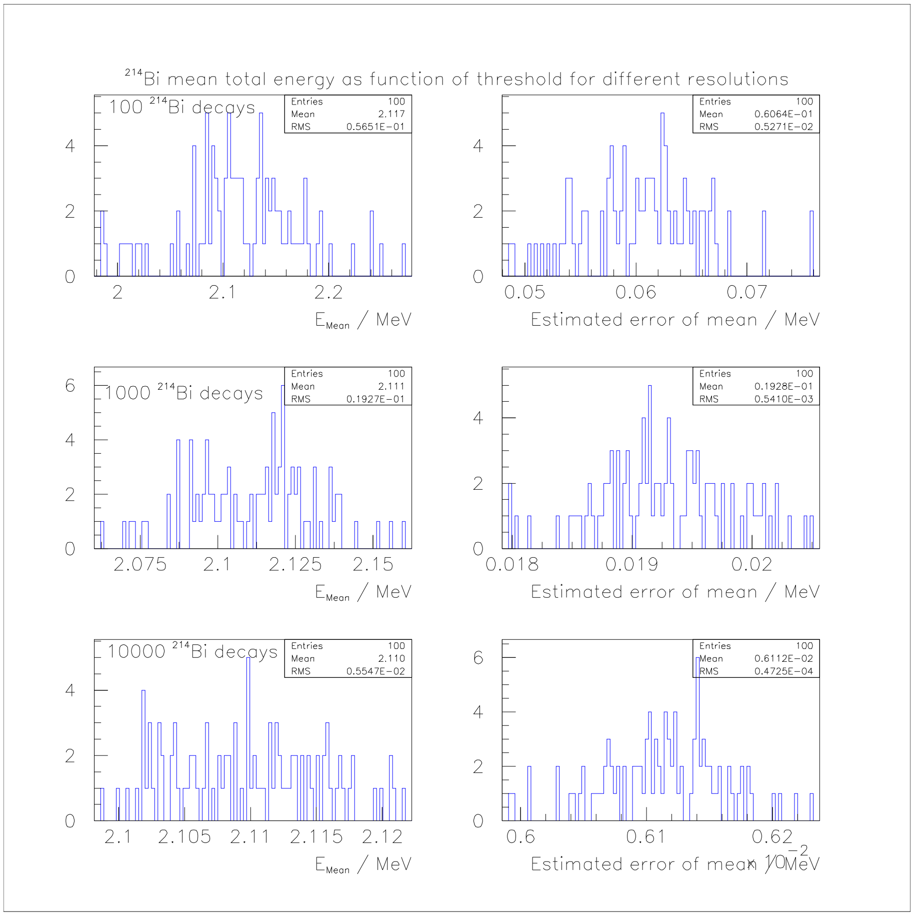

Figure 2: Repeated analysis of the ^{214}Bi mean energy for different numbers of decays. The left panels show the distributions of the calculated mean energies and their variance. On the right hand side the distribution of the individual error estimates is presented. The fluctuation of the means, shown as RMS in the left panels, is on average, equal to within 10% to the mean of the individual error estimates. This demonstrates that our error estimated is a reasonable measure of the fluctuation of the mean energy.

resolutions. The results of this calculation are shown in figure 3. We see that the mean energy is rather insensitive to the resolution folding. This is not surprising as the underlying Bi energy distribution is much broader than the resolution smearing. The mean energy is also rather inert against

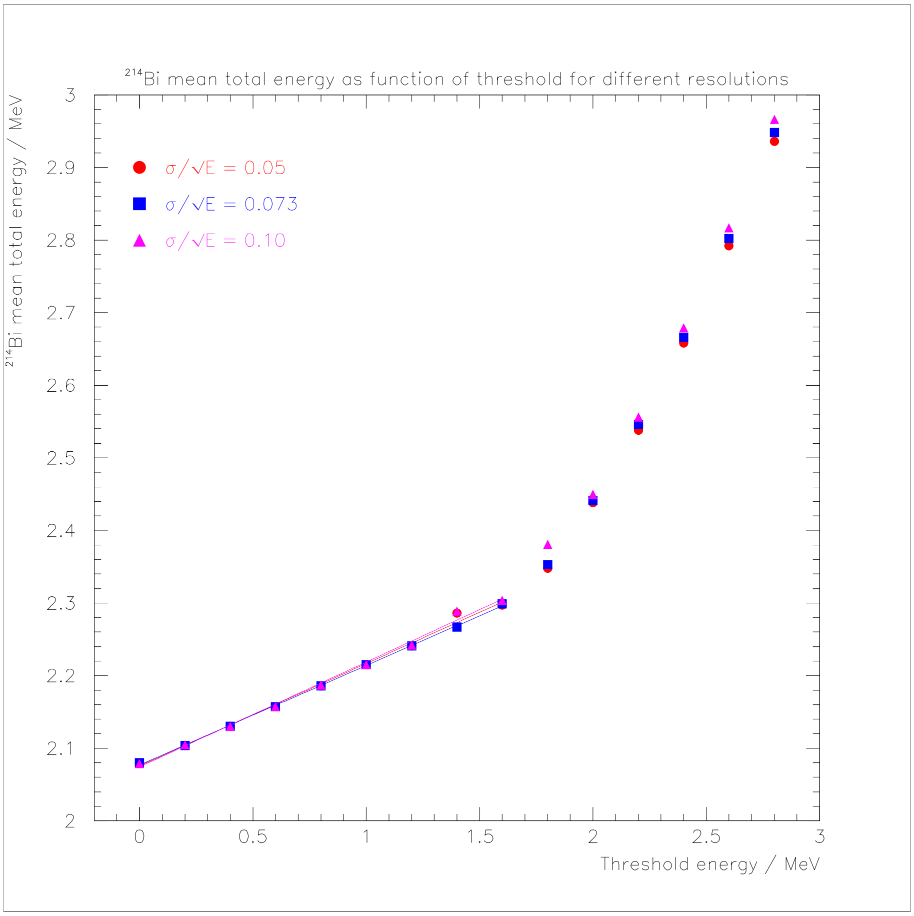

Figure 3: Dependence of the mean energy on the threshold cut and the energy resolution.

the threshold cut, as long as the threshold is below 1.8 MeV. As soon as the threshold starts to cut into the peak the energy determination starts to become more and more biased by the threshold. From the linear fits at low energies we see that a 5% uncertainty in the threshold translates into a 0.7% uncertainty in the mean energy.

3 Data Analysis

Event reconstruction has been performed using the AKat charge based vertex and energy fitter. In this analysis only the delayed energy has been corrected down according to the energy calibration runs. This correction amounts to a down-shift of the delayed alpha peak by a factor of 0.93. No such correction has been applied to the prompt signal. The analysis of the mean Bi energies is therefore based solely on the detected charge.
In order to see how well the calculated shape matches the measured one we may directly compare two distributions. This is done in in figure 5. The measured data (red) was selected using the following cuts to the 30.8045 days of data available to us:

1. There are two sequential hits within $40\ \mu s \leq\ \Delta T\ \leq\ 1000\ \mu s$.
2. Prompt energy deposit, E_p, satisfies $E_p\ \geq\ 1.6\ MeV$
3. Delayed energy deposit, E_d, satisfies $0.3\ \leq\ E_d\ \leq\ 1.0\ MeV$
4. Spherical radius of the prompt event, $R_p\ \leq\ 5.0\ m$
5. The prompt and delayed spherical radii are within 1.5 m: $|R_p - R_d| \leq 1.5\ m$

Figure 4 depicts the distributions of the parameters listed above. From the fits to these data we see:

1. The energy of the delayed alpha peak corresponds with 0.593 ± 0.003 MeV to a quenching factor of 12.96 ± 0.07.
2. The mean correlation time between prompt and delayed sub-event is $\tau\ =\ 227 \pm 29\ \mu s$. The tabulated mean live time of ^{214}Po is $237\ \mu s$.
3. From the time distribution we estimate a signal to background ratio of 4.7 ± 0.6 for this data set.

In figure 5 we show the direct comparison of measured and calculated energy distribution. This data thus tests the energy scale properly averaged over 30 days and over the fiducial volume up to $R = 5\ m$. The measured distribution has a mean energy of $\langle E_{Exp} \rangle\ =\ 2.366 \pm 0.016\ MeV$. The calculated distribution yields $\langle E_{Mod} \rangle\ =\ 2.299 \pm 0.002^{stat}\ MeV$. In this averaged data

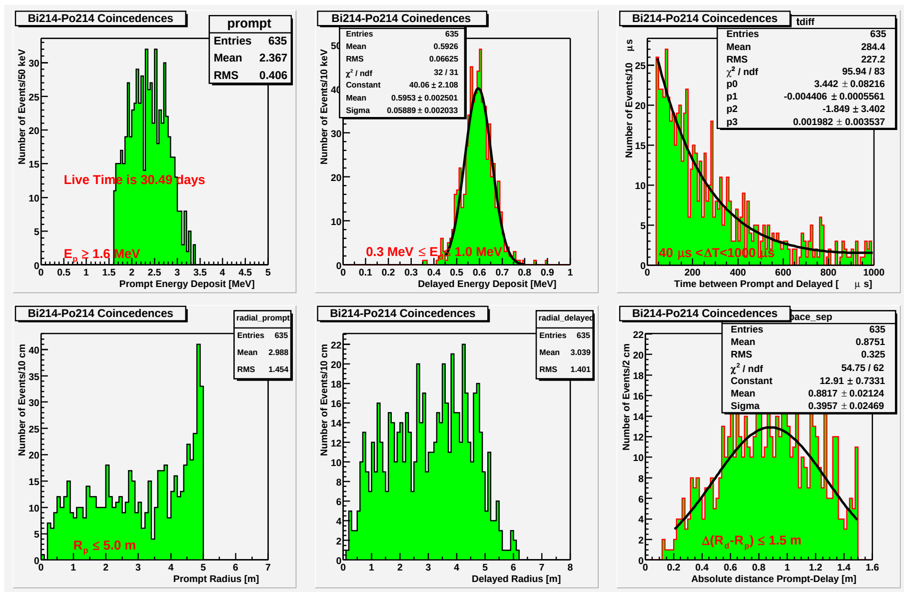

Figure 4: Distribution of the parameters used to isolate ^{214}Bi decays.

set we thus find that the measured mean energy is $2.9 \pm 0.02\%$ larger than the expectation. Based on the error estimate defined by equation 2 the measured mean energy is determined with 0.7% precision. The measured and calculated distributions shown in figure 5 have a width, s_E, of 0.406 MeV and 0.359 MeV, respectively.

We now perform a similar analysis for spherical shells to probe the off-axis energy calibration. Figure 6 shows a comparison of measured and calculated mean energies in Bi decays for spherical shells. The thickness of the shells (dR) has been chosen so that roughly the same number of counts is contained in each spectrum. No spectrum contains less than 100 events. The number of available Bi decays increases dramatically as vertex positions near the balloon are allowed. For these data a large low energy event excess is noted. This could be due to random background or due to the fact that a real Monte Carlo simulation is needed there to account for energy loss out of the active medium. This test is therefore only applicable up to a radius of 5 m. However, this is still very much "off-axis" and close to the border of the fiducial cut. To demonstrate the low energy contamination of near balloon vertices we

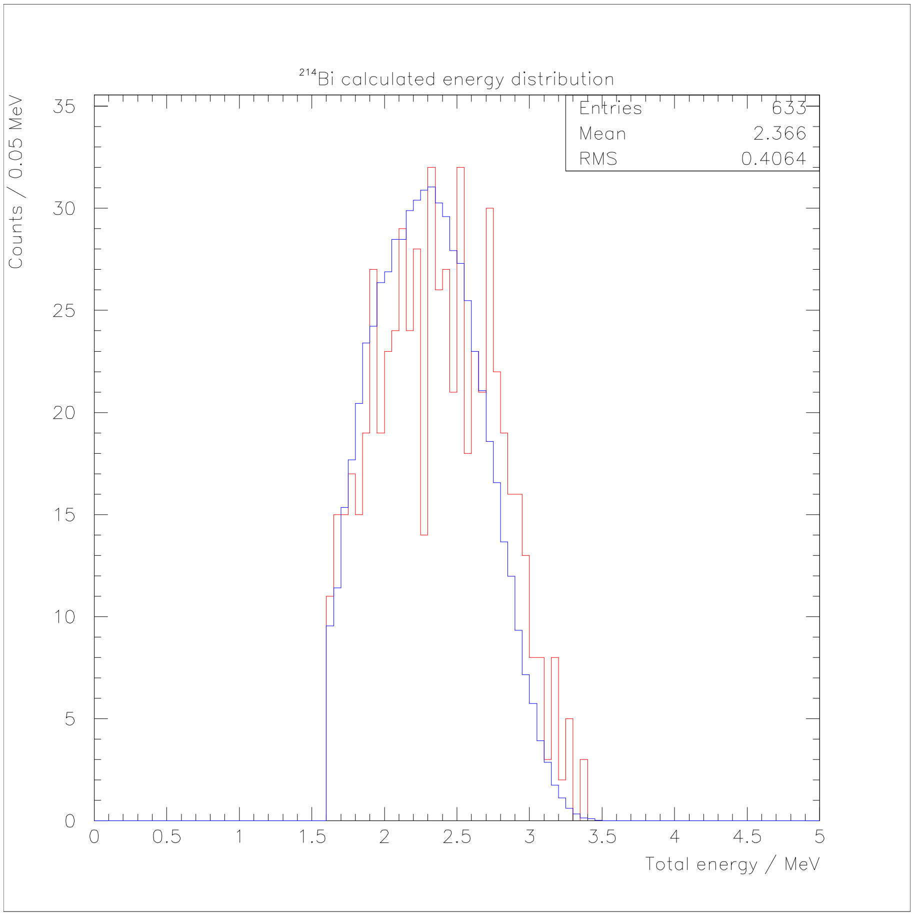

Figure 5: Comparison of measured (red) and calculated (blue) total energy release in ^{214}Bi beta decays. The calculated distribution is based on the KamLAND Monte Carlo event generator, which outputs particle kinetic energy without detector dependent corrections. The calculated distribution has been folded with an energy resolution of $\sigma(E)/\sqrt{E} = 0.073$.

repeated the position dependent energy test but now with a threshold value of 1.8 MeV for the region centered around 5.25 m. The resulting data is shown in figure 7. Indeed the mean energy moves closer to the expectation. The lower panels in both figures 6 and 7 show the estimated signal to background ratios for the R-dependent energy analysis. From this we clearly see that the point at R=5.25 m is plagued by a considerable amount of background. It

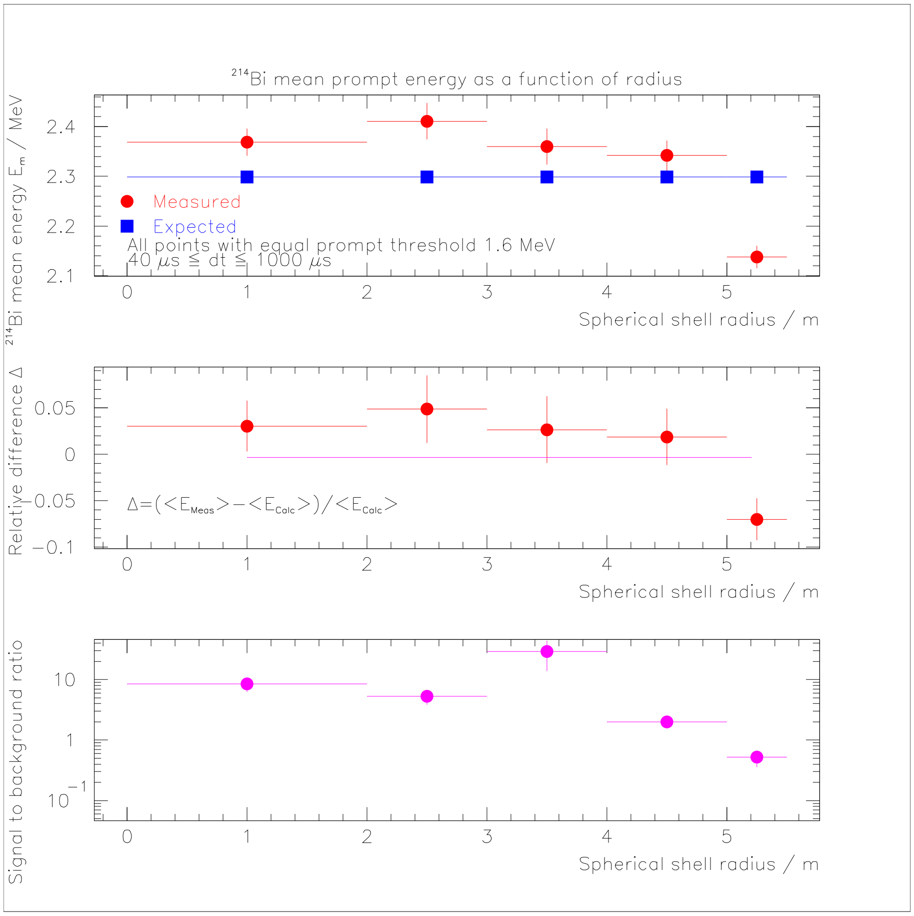

Figure 6: Comparison of measured (red) and calculated (blue) mean total energies for ^{214}Bi. All point correspond to an energy threshold of 1.6 MeV. The lower part shows the associated signal to background ratios.

should therefore not be included in the analysis.

4 Conclusion

From the energy analysis displayed in figures 6 and 7 we see that measured and expected mean energies in Bi decays agree with each other within 3.0% throughout the detector. The measured energy is actually larger than the

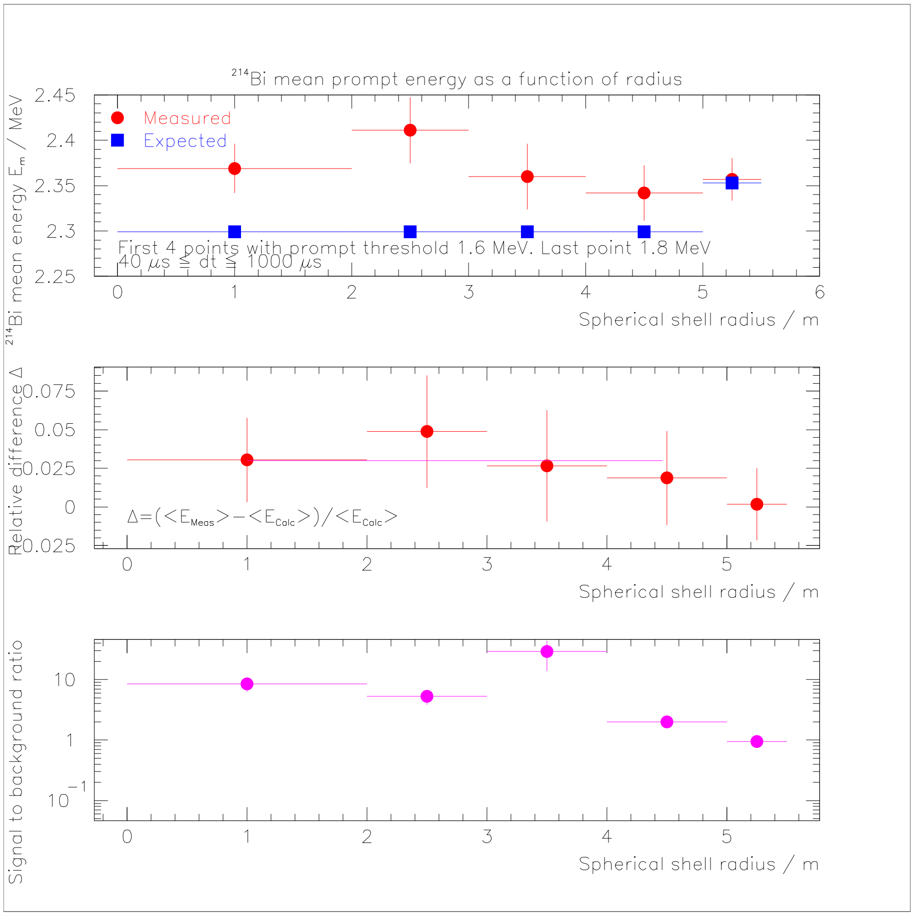

Figure 7: Comparison of measured (red) and calculated (blue) mean total energies for ^{214}Bi. For all but the 5.25 m point the energy thresholds were 1.6 MeV. To improve the signal to background ratio of the 5.25 m point an energy threshold of 1.8 MeV was used. The lower part shows the associated signal to background ratios.

expected one. The uniformity of the mean energy determined for the detector sub-shells is with ±1.6% quite small. This value agrees with the value obtained with sources moved along the z-axis.