

Effective Neutron Capture Cross Sections

Andreas Piepke
Department of Physics and Astronomy
University of Alabama

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Abstract

Neutron capture cross sections are conveniently tabulated in reference books or on the web. However, if neutron activation at nuclear reactors is to be performed we have to take into account that the neutron flux there is not mono-energetic but instead spread over a wide spectral range. An appropriate average of the energy dependent capture cross section has hence to be performed to obtain reliable cross sections which are consistent for different isotopes.

1 Reactor Neutron Spectrum

Parameterizations of reactor neutron spectra can be found in many text books as e.g. reference [1]. Here only a brief description: neutrons emitted in the fission of ^{235}U have kinetic energies between 0 and 25 MeV. In the following we will neglect the contribution of those fast fission neutrons to the averaged capture cross section.

The neutrons are slowed down by collisions with the moderator atoms until they reach thermal equilibrium. The neutrons slowing down through elastic scattering show an energy distribution $\Phi_{epi}(E)$ which varies approximately by E^{-1} :

$$\Phi_{epi}(E) \sim E^{-1} \tag{1}$$

This parameterization of the epi-thermal neutron spectrum holds for moderators showing negligible neutron absorption. The epi-thermal flux normalization depends on the reactor, its fuel composition and the proximity of the irradiation site to the reactor core. It needs to be known in order to interpret activation data.

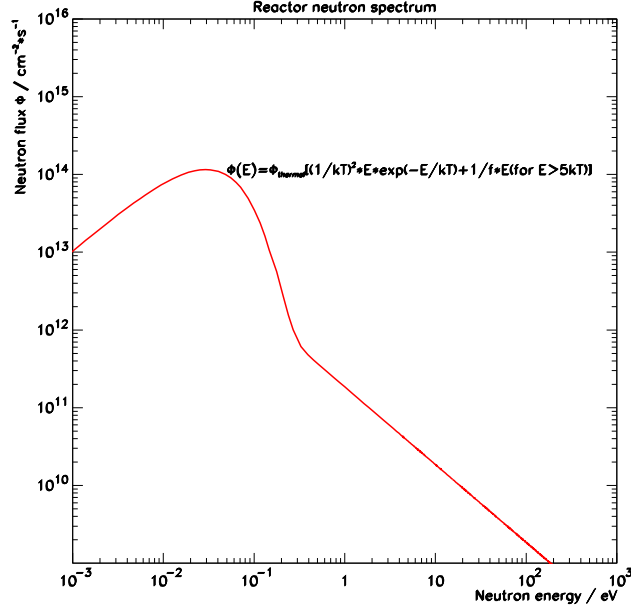


Figure 1: Reactor neutron spectrum.

Neutrons eventually reach thermal equilibrium. At these energies the spectrum $\Phi_{th}(E)$ is described by a Maxwell-Boltzmann distribution:

$$\Phi_{th}(E) = \frac{E}{(kT_m)^2} \exp\left(-\frac{E}{kT_m}\right), \quad (2)$$

with k being the Boltzmann constant and T_m the temperature of the moderator. The energy distribution obviously depends on the temperature of the moderator. The research reactors at Oak Ridge National Laboratory and MIT have approximately $T_m = 340$ K. Figure 1 depicts the reactor neutron spectrum.

The neutron spectrum $\Phi(E)$ is usually parameterized as:

$$\Phi(E) = \begin{cases} \Phi_{th}(E) + \frac{1}{f}\Phi_{epi}(E) & \text{for } kT_m > 5 \text{ eV} \\ \Phi_{th}(E) & \text{else} \end{cases} \quad (3)$$

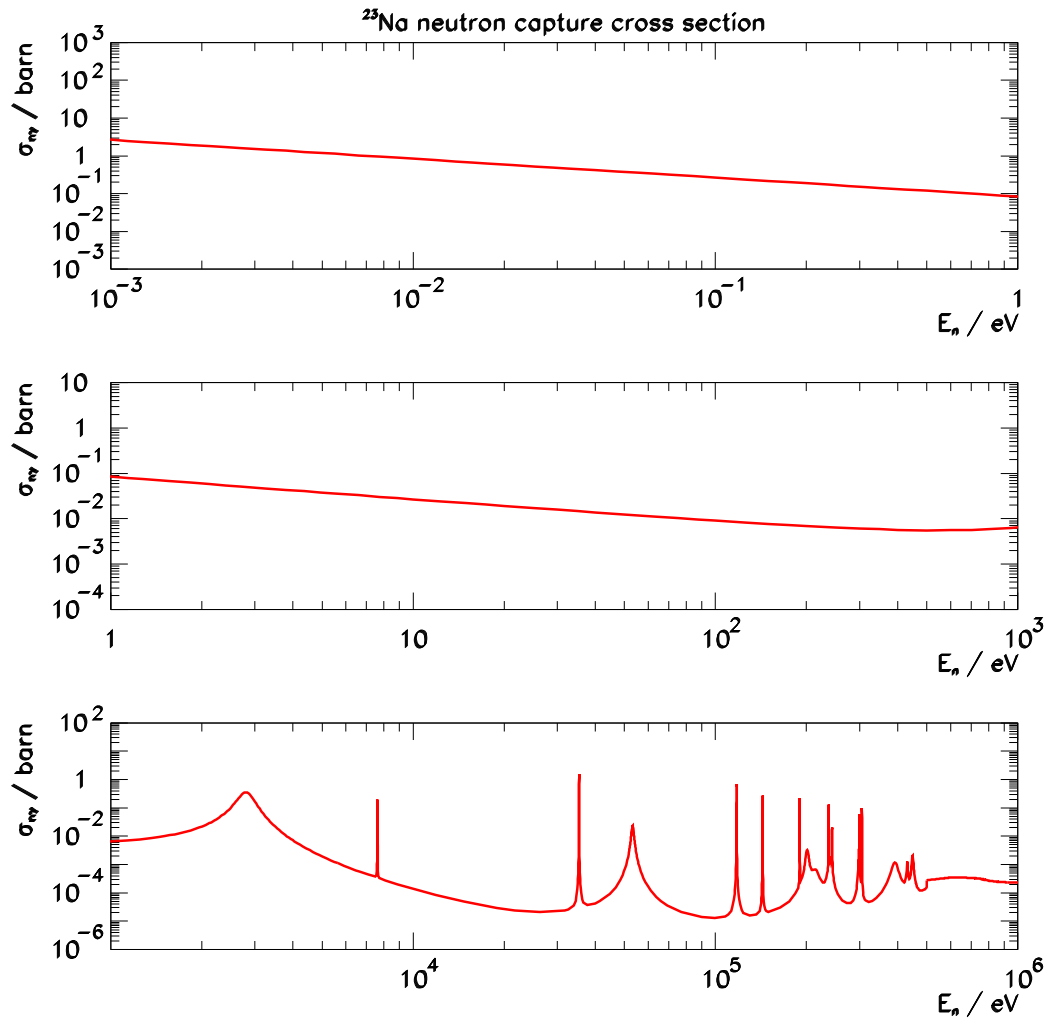


Figure 2: ²³Na (n,γ)-cross section.

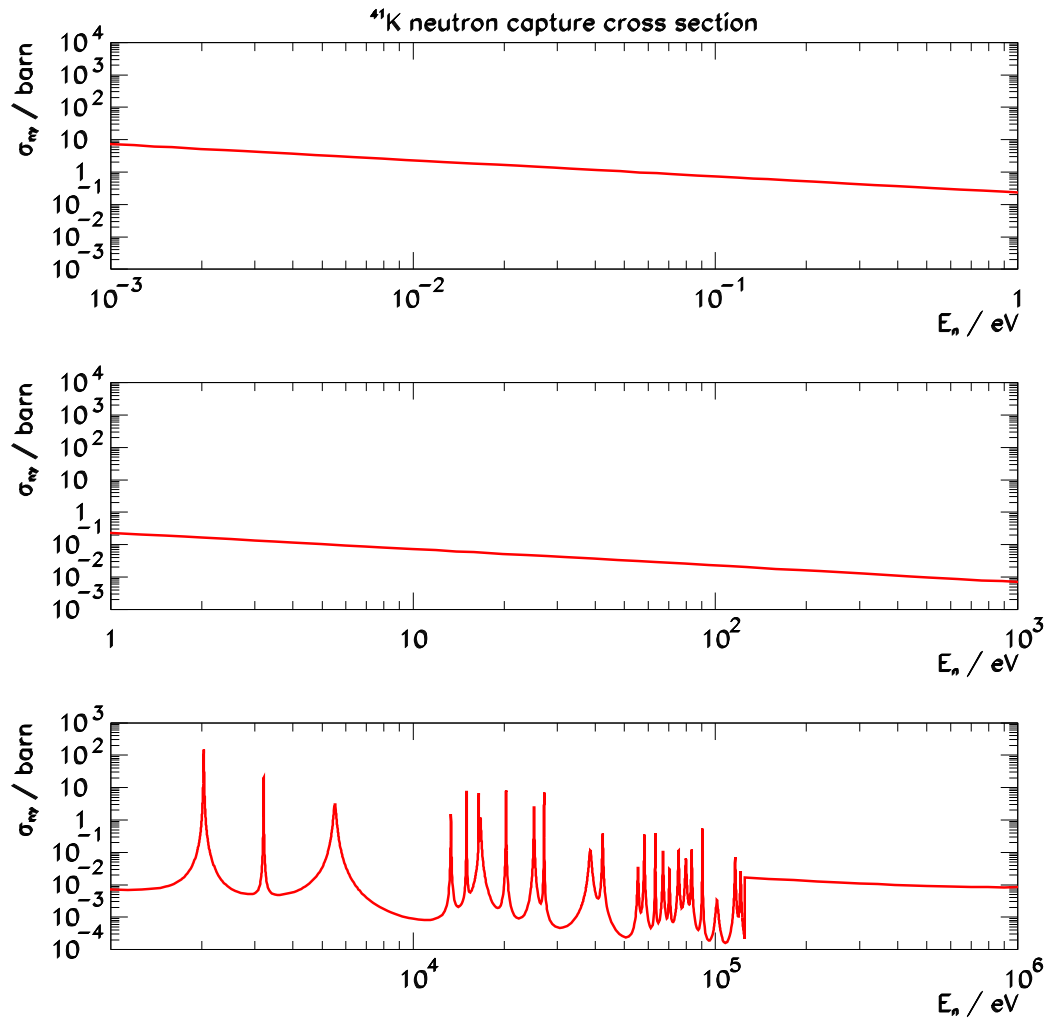


Figure 3: ^{41}K (n, γ)-cross section.

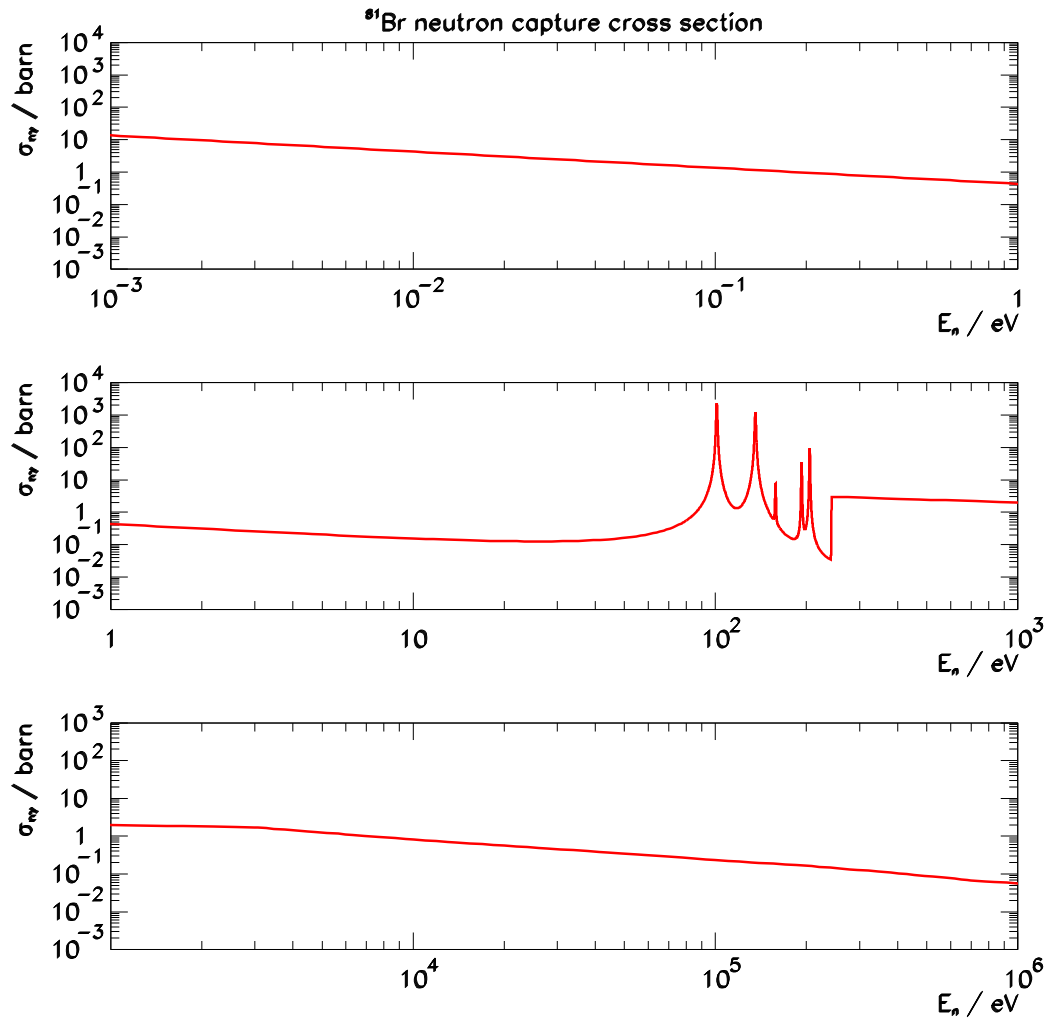


Figure 4: ^{81}Br (n, γ)-cross section.

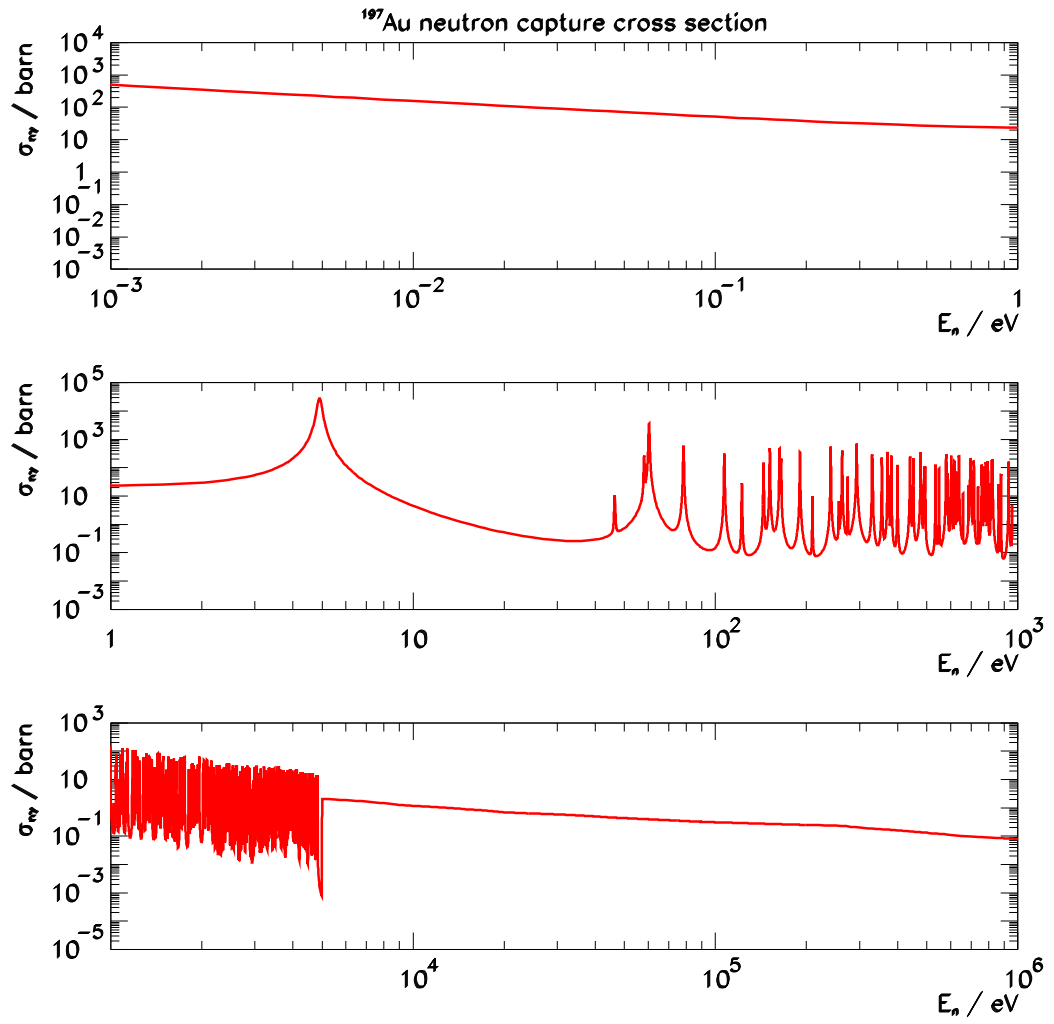


Figure 5: ¹⁹⁷Au (n,γ)-cross section.

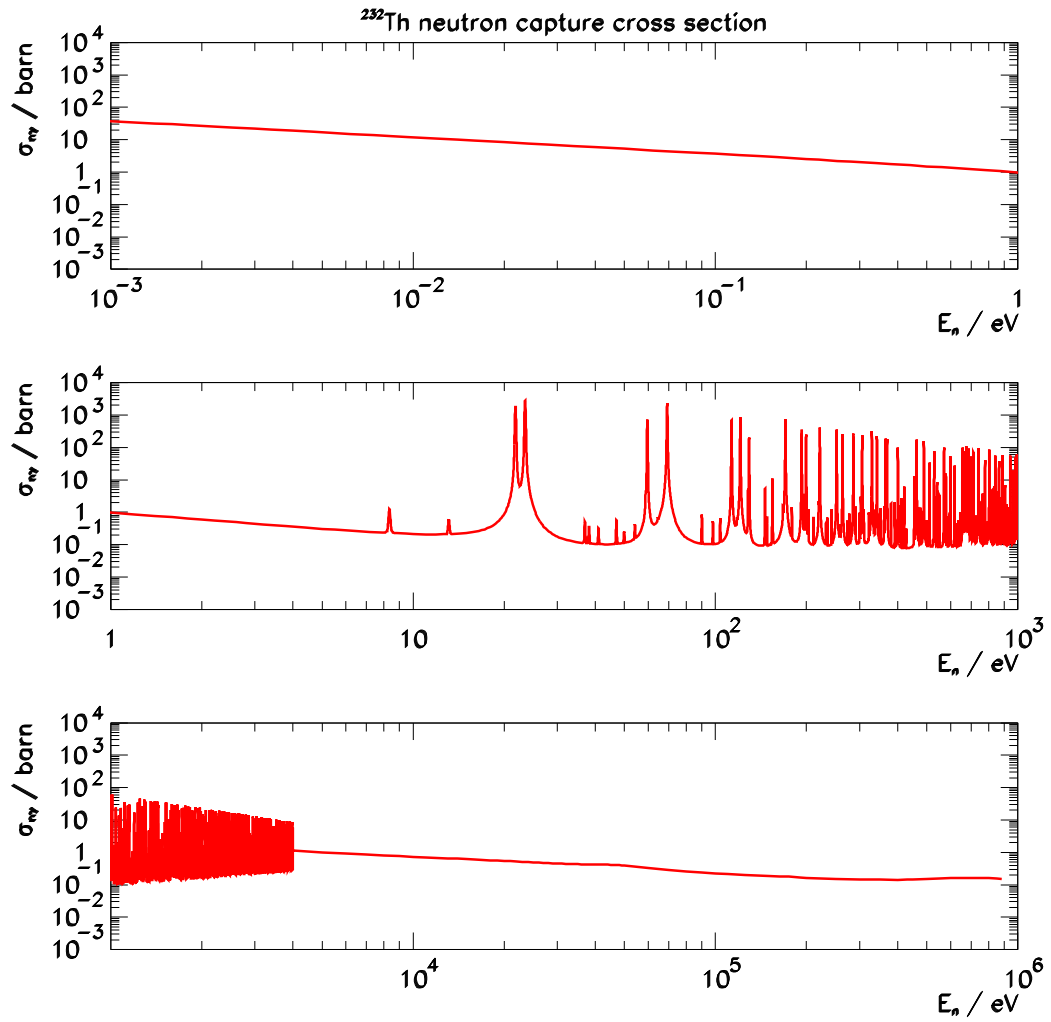


Figure 6: ²³²Th (n,γ)-cross section.

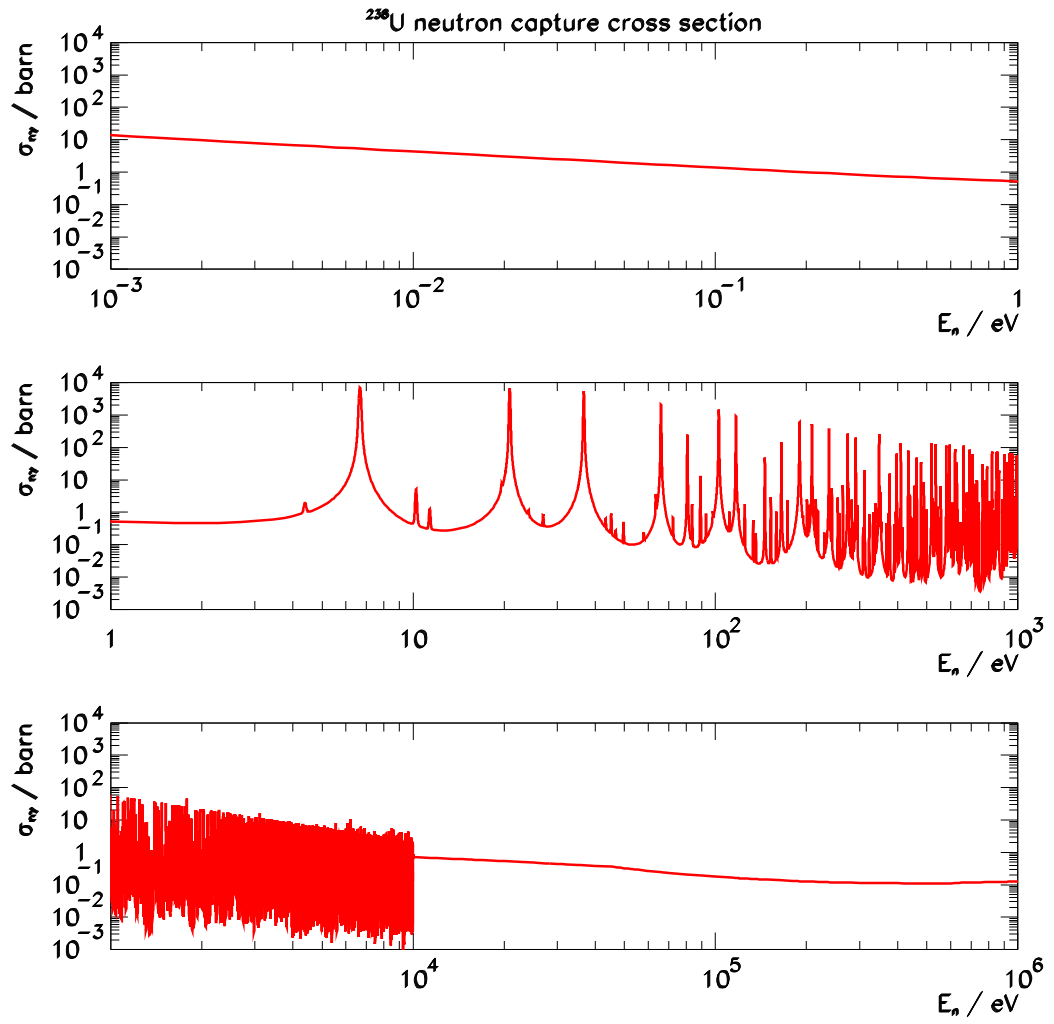


Figure 7: ^{238}U (n, γ)-cross section.

2 Capture Cross Section

Energy dependent neutron capture cross sections are known for most isotopes. They may be down-loaded from the WWW e.g. from “<http://atom.kaeri.re.kr/>”, the web site of the “Korea Atomic Energy Research Institute”. There the ENDF (Evaluated Nuclear Data Files) data set is used. Figures 2 through 7 show the cross section for various isotopes of interest. Isotopes like ^{238}U show substantial capture cross sections for epi-thermal neutrons. A realistic averaged cross section $\langle\sigma\rangle$ is given by:

$$\langle\sigma\rangle = \frac{\int \sigma(E) \Phi(E) dE}{\int \Phi(E) dE}. \quad (4)$$

The average cross section may be separated in a thermal and epi-thermal integral (the latter one is called the resonance integral):

$$\begin{aligned} \langle\sigma_{th}\rangle &= \frac{\int_{E_l}^{E_m} \sigma(E) \Phi(E) dE}{\int_{E_l}^{E_m} \Phi(E) dE} \\ \langle\sigma_{epi}\rangle &= \frac{\int_{E_m}^{E_h} \sigma(E) \Phi(E) dE}{\int_{E_m}^{E_h} \Phi(E) dE} \end{aligned} \quad (5)$$

For $E_l = 10^{-3}$ eV, $E_m = 5$ eV, and $E_h = 10^6$ eV these averaged cross sections may be found tabulated in various books. $\langle\sigma_{th}\rangle$ is usually tabulated for $T=293.6$ K.

Let the integrated neutron flux Φ_{th} and Φ_{epi} be given as:

$$\begin{aligned} \Phi_{th} &= \int_{E_l}^{E_m} \Phi(E) dE \\ \Phi_{epi} &= \int_{E_m}^{E_h} \Phi(E) dE \\ f &= \frac{\Phi_{th}}{\Phi_{epi}} \end{aligned} \quad (6)$$

Using these definitions, the total integral of the product of cross section and flux can be expressed in terms of the (tabulated) averaged cross sections and the total flux integral:

$$\int_{E_l}^{E_h} \sigma(E) \Phi(E) dE = \Phi_{th} \cdot (\langle\sigma_{th}\rangle + \frac{1}{f}\langle\sigma_{epi}\rangle) = \Phi_{th} \cdot \sigma_{eff} \quad (7)$$

Using this approximation the activation may be described by two numbers: integral thermal neutron flux Φ_{th} and epi-thermal flux suppression factor f .

3 Averaged Cross Sections

Above system of definitions is now used to calculate averaged cross sections for various isotopes. Room temperature and a more realistic $T=340$ K is used. To show that these numbers make sense they are compared to tabulated values whenever those are available. The table below gives a compilation of the averaged cross section. All cross sections are in *barn*. Tabulated data has been taken from “<http://www.nes.ruhr-uni-bochum.de/CoN/index.html>”. While the thermal cross sections evaluated at room temperature are typically

Isotope	Tabulated [barn]			This evaluation [barn]		
	σ 293.6 K	$\langle\sigma_{th}\rangle$ 293.6 K	$\langle\sigma_{epi}\rangle$	$\langle\sigma_{th}\rangle$ 293.6 K	$\langle\sigma_{th}\rangle$ 340 K	$\langle\sigma_{epi}\rangle$
²³ Na	0.531	0.471	0.311	0.467	0.434	0.349
⁴¹ K	1.459	1.294	1.58	1.298	1.206	1.664
⁵⁰ Cr	15.92	14.12	7.38	14.13	13.13	8.30
⁵⁸ Fe	1.300	1.153	1.357	1.020	0.948	1.561
⁵⁹ Co	37.18	32.96	75.51	33.07	30.72	76.68
⁶⁴ Zn				0.677	0.629	1.564
⁸¹ Br	2.690	2.385	46.63	2.385	2.217	51.03
¹⁹⁷ Au				88.02	81.99	1566.7
²³² Th	7.400	6.532	84.35	6.567	6.095	86.91
²³⁸ U	2.717	2.414	278.1	2.421	2.251	278.3

20% larger than the properly averaged cross section (at higher temperature), the effective cross section σ_{eff} evaluated for $f = 33$ shows only about 7% discrepancy. However, as it is straight forward to include these correction we should not neglect them. Using a consistent set of cross sections in all laboratories involved in this work will help to ensure that we derive compatible results.

References

- [1] D. De Soete, R. Gijbels and J. Hoste, “Neutron Activation Analysis”, Wiley-Interscience, 1972