Reactor Power Systematic Error for KamLAND

Z. Djurcic
Department of Physics and Astronomy,
University of Alabama,
Tuscaloosa, AL

October 31, 2003

Abstract

In this note I examine the influence of systematic error of thermal power to the KamLAND anti-neutrino rate from the nuclear power reactors. At present, the systematic error assigned to the thermal power is conseravtively taken as 2% from the regulatory specification for safe reactor operation as reported in the First Results From KamLAND. Closer investigation shows this error may be as low as 0.5%.

1 Information Source

The informations and suggestions used here are partially obtained through private communication [1]. This communication has nothing to do with "theta13" project involving some of KamLAND colleagues aiming to use Diablo Canyon as anti-neutrino source.
The values of reactor power systematic errors used in japanese electric companies are obtained from Suekane-san. According to Suekane-san, the values (1.8% for Boiling Water Reactor (BWR) and 1.6% for Pressurazed Water Reactor (PWR) type) are mostly from standard deviation of the accuracy of the measurement of a flow rate. To be conservative, 2% values were used for the PRL.
In all calculations performed below I use the basic reactor data provided by

RCNS in [2].

2 Error Propagation

The rate of detected anti-neutrino events n_ν^i from i-th nuclear reactor in KamLAND is:

$$n_\nu^i = \frac{1}{4\pi L_i^2}\,\frac{W_{th}^i}{E_f}\,N_p\,\epsilon\,\sigma_f \tag{1}$$

where L_i is the distance of i-th nuclear reactor to KamLAND detector, W_{th}^i is the i-th reactors thermal power, N_p is the number of protons, ϵ is efficiency for the event detection, σ_f is the effective cross section per fission, and E_f is average energy release per fission.
This can be written as:

$$n_\nu^i = k\,\frac{W_{th}^i}{L_i^2} \tag{2}$$

where

$$k = \frac{1}{4\pi}\,\frac{N_p\,\epsilon\,\sigma_f}{E_f} \tag{3}$$

Therefore, total contribution to the detected number of events from all n_r nuclear reactors in KamLAND will be

$$n_\nu^{total} = k\,\sum_{i=1}^{n_r}\frac{W_{th}^i}{L_i^2} \tag{4}$$

Systematic errors of thermal powers of different nuclear reactors can be correlated or uncorrelated.
If there is no correlation, standard deviation of detected number of anti-neutrino events is determined as

$$\sigma(n_\nu^{total}) = \left[|\frac{\partial n_\nu^{total}}{\partial W_{th}^1}|^2\,\sigma^2(W_{th}^1)+|\frac{\partial n_\nu^{total}}{\partial W_{th}^2}|^2\,\sigma^2(W_{th}^2)+...+|\frac{\partial n_\nu^{total}}{\partial W_{th}^{n_r}}|^2\,\sigma^2(W_{th}^{n_r})\right]^{1/2} =$$

$$= \left[(\mathrm{k}\,\tfrac{1}{L_1^2}\,\sigma(W_{th}^1))^2 + (k\,\tfrac{1}{L_2^2}\,\sigma(W_{th}^2))^2 + ... + (k\,\tfrac{1}{L_{n_r}^2}\,\sigma(W_{th}^{n_r}))^2\right]^{1/2} =$$

$= \left[(k \frac{W_{th}^1}{L_1^2} \frac{\sigma(W_{th}^1)}{W_{th}^1})^2 + (k \frac{W_{th}^2}{L_2^2} \frac{\sigma(W_{th}^2)}{W_{th}^2}))^2 + ... + (k \frac{W_{th}^{n_r}}{L_{n_r}^2} \frac{\sigma(W_{th}^{n_r})}{W_{th}^{n_r}})^2 \right]^{1/2} =$

$= \left[(n_\nu^1 \frac{\sigma(W_{th}^1)}{W_{th}^1})^2 + (n_\nu^2 \frac{\sigma(W_{th}^2)}{W_{th}^2})^2 + ... + (n_\nu^{n_r} \frac{\sigma(W_{th}^{n_r})}{W_{th}^{n_r}})^2 \right]^{1/2} =$

$= \left[\sum_{i=1}^{n_r} (n_\nu^i \ \delta_{th}^i)^2 \right]^{1/2} (5)$

where δ_{th}^i now represent the systematic error on the thermal power of i-th nuclear reactor, that is

$$\delta_{th}^i = \frac{\sigma(W_{th}^i)}{W_{th}^i} \tag{6}$$

If there is a correlation between systematic errors of thermal powers of different reactors, standard deviation of detected number of anti-neutrino events is determined as

$$\sigma(n_\nu^{total}) = \left[\sum_{i=1}^{n_r} \sum_{j=1}^{n_r} |\frac{\partial n_\nu^{total}}{\partial W_{th}^i}||\frac{\partial n_\nu^{total}}{\partial W_{th}^j}|\alpha_{ij}\sigma(W_{th}^i)\ \sigma(W_{th}^j) \right]^{1/2} =$$

$= \left[\sum_{i=1}^{n_r} \sum_{j=1}^{n_r} n_\nu^i \ n_\nu^j \alpha_{ij} \frac{\sigma(W_{th}^i)}{W_{th}^i} \frac{\sigma(W_{th}^j)}{W_{th}^j} \right]^{1/2} =$

$= \left[\sum_{i=1}^{n_r} \sum_{j=1}^{n_r} n_\nu^i \ n_\nu^j \alpha_{ij} \ \delta_{th}^i \ \delta_{th}^j \right]^{1/2} (7)$

where α_{ij} is correlation coefficient between the thermal powers of reactors i and j.

An example: Kashiwazaki at nominal power. There are 7 reactors of BWR type.

Number	Code	Dist.(m)	Type	Company	Reactor	Power(MWth)	Power(MWe)
47	TKKK1	159165.2	BWR	Tokyo	KashiwazakiKariwa_1	3293	1100
48	TKKK2	159286.5	BWR	Tokyo	KashiwazakiKariwa_2	3293	1100
49	TKKK3	159396.2	BWR	Tokyo	KashiwazakiKariwa_3	3293	1100
50	TKKK4	159598.7	BWR	Tokyo	KashiwazakiKariwa_4	3293	1100
51	TKKK5	160680.4	BWR	Tokyo	KashiwazakiKariwa_5	3293	1100
52	TKKK6	160580.5	BWR	Tokyo	KashiwazakiKariwa_6	3926	1356
53	TKKK7	160459.9	BWR	Tokyo	KashiwazakiKariwa_7	3926	1356

Table 1: Basic reactor data for Kashiwazaki nuclear reactors at nominal power extracted from [2].

To see how this simple math works assume $L_1 = L_2 = L_3 = L_4 = L_5 = L_6 = L_7$, and $W_{th}^1 = W_{th}^2 = W_{th}^3 = W_{th}^4 = W_{th}^5 = W_{th}^6 = W_{th}^7$, that is close to reality as may be seen in the table. Therefore $n_\nu^1 = n_\nu^2 = n_\nu^3 = n_\nu^4 = n_\nu^5 = n_\nu^6 = n_\nu^7$. Then, if there is no correlation

$$\sigma(n_\nu^{total}) = \left[7\ (n_\nu^1\ \frac{\sigma(W_{th}^1)}{W_{th}^1})^2\right]^{1/2} \tag{8}$$

and $n_\nu^{total} = 7\ n_\nu^1$, what gives

$$\frac{\sigma(n_\nu^{total})}{n_\nu^{total}} = \frac{1}{7\ n_\nu^1}\ \left[7\ (n_\nu^1\ \delta_{th}^1)^2\right]^{1/2} = \frac{\delta_{th}^1}{\sqrt{7}} \tag{9}$$

One may see that relative error in anti-neutrino rate, caused by thermal power uncertainities, goes down as $1/\sqrt{(number\ of\ reactors)}$.
However, if there is correlation (assume maximal correlation with $\alpha_{ij} = 1$), then for Kashiwazaki this results in

$$\sigma(n_\nu^{total}) = \left[\sum_{i=1}^{7}\ \sum_{j=1}^{7} n_\nu^i\ n_\nu^j \alpha_{ij}\ \delta_{th}^i\ \delta_{th}^j\right]^{1/2} = \left[49\ (n_\nu^1\ \delta_{th}^1)^2\right]^{1/2} = 7\ n_\nu^1\ \delta_{th}^1 \tag{10}$$

and

$$\frac{\sigma(n_\nu^{total})}{n_\nu^{total}} = \frac{1}{7\ n_\nu^1}\ 7\ (n_\nu^1\ \delta_{th}^1) = \delta_{th}^1 \tag{11}$$

Here, the relative error does not depend on the number of reactors.
For BWRs we have $\delta_{th}^i = 1.8\%$. Without correlation one gets 0.68% for the relative uncertainty in the anti-neutrino rate, while with maximal correlation one finds 1.8% for the same rate starting from 1.8% uncertainty in the thermal power in Kashiwazaki example. Of course, taking 2% for all reactor thermal power errors as in PRL will produce 2% error in the anti-neutrino rate.

Perfoming the same exercise as in the example above with all 76 reactors in KamLAND's neigbourhood, taking 1.8% as thermal power systematic error for BWRs, 1.6% for PWRs and 1.8% for any other type (negligible contribution to the anti-neutrino rate), one obtains 1.72% for the relative uncertainty in the anti-neutrino rate at KamLAND in the case of maximal correlation of the reactor errors. This number goes down to only 0.32% in the case where all reactor errors are uncorrelated.

3 How Much Is Systematic?

In Palo Verde [3], [4] calorimetric methods (3 PWRs were used) based on measurement of temperature and water flow rate in the secondary cooling loop provided a power determintion with 0.7% uncertainty. Calculation from secondary calorimetry showed that most of uncertainty comes from the secondary flow estimate, notably the diameter of the venturi pipe used.
The problem is to understand, when an error is assigned to each reactor, whether the above errors sum in quadrature or linearly, i.e. whether they are random or systematic.
Again, in Palo Verde, the intrinsic errors are considered systematic, since all 3 reactor units use identical algorithms.

In most nuclear power plants [6], operators obtain a continuous indication of core thermal power from nuclear instruments that provide a measurement of neutron flux. The nuclear instruments must be periodically calibrated to counteract the effects of changes in flux pattern, fuel burnup, and instrument drift. Steam plant calorimetry, which is the process of performing a heat balance around the nuclear steam supply system (called a calorimetric), is used to determine core thermal power and is the basis for the calibration. The differential pressure across a venturi installed in the feedwater flow path is a key element in the calorimetric measurement. Some plants use this calorimetric value directly to indicate thermal power; the nuclear instruments are used as anticipatory indicators for transients and for reactivity adjustments made with the control rods.

Thermal power is expressed as mass flow rate to the steam generator times the change in heat across the entering feedwater and the exiting steam

$$W_{th} = m\ c_p\ h \tag{12}$$

where W_{th} is thermal power, m is mass flow rate of the feedwater to the steam generator measured by the UFM (Ultrasonic Flow Meter) [5], c_p is constant for water, and h is the energy (enthalpy) rise across the steam generator on the feedwater side. For example, for Diablo Canyon Power Plant (DCPP) aproximate values are m=14963290 lbm/hr, h=782.9 btu/lbm. Please note, all values are in US English Units.
The heat that is transferred to the steam generator comes from the primary

plant. There is a reactor to add heat to the water, pumps that add heat, but the pipes etc. are insulated so there is a heat loss there. Consequently, steam genertor power is not the same as reactor core power for a PWR. A BWR does not have all those losses. It is still the same equation only the heat rise is across the reactor not the steam generator.

Unfortunately, it is not yet understood at what confidence level the power errors are being reported. There is an early standard, and a later one which most US plants are lincensed to when report the thermal power uncertainty. For example, DCPP was lincensed to the earlier standard entitled BELOCA (Best Estimate Loss of Cooling Accident). That means that the uncertainty allowed is slightly less than 2% for DCPP. SONGS (San Onofre Nuclear Generation Station) is licensed to 2% uncertainty. Using the AMAG Crossflow Ultrasonic Flow Measurement device, they are allowed to recapture uncertainty [6]. This uncertainty recapture can can be as high as 1.6%. Thus this implies that the total power uncertainty calculation must be less than or equal to 0.4%. These are all related to ECCS (emergency core cooling system) margin, or ability to cool the reactor core in case of an accident. When the US licenses a power plant using an uncertainty, it is essentially stating that if the uncertainty was at the top of its band the core would still be able to be cooled in the case of an accident starting at the high end of the power band.
Therefore, if we know the power level we are at and we have an uncertainty that does not exceed the licenced level, we can protect the core from the catastrophic failure. As an example suppose that a power plant is licensed for 100% power with a 2% uncertainty at a core rating of 3500 MWt. If the plant was operating at 3570 MWt core power at the start of an accident, ECCS system is designed to protect the power plant.

Assuming only systematic part of the thermal power uncertainty can be correlated when calculate anti-neutrino rate, it is important to estimate which part of the uncertinty is systematic. Again according to [1], the flow error is systematic but is used as a random error. The pressure and temperature measurements used are also biased. Then when calculate the enthalpy of the steam leaving the steam generator there is another systematic error there.
In conclusion, out of the 2% uncertainty that is allowed, perhaps as much as 0.5% of the total uncertainty is systematic errors.
By using the upgraded UFM and upgrading the instrumentation, one could

end up with an error of 0.4% in determining reactor power, but that would include a systematic error that reactor people treat as random. The systematic error at that point should be no more than about 0.1%.

4 Importance to KamLAND

Assuming that above is applicable to KamLAND, then 1/4 of the errors assigned to the thermal powers of the japanese nuclear reactors is to be systematic. Only those systematic errors could be correlated. Starting from eq. (7)

$$\sigma(n_\nu^{total}) = \Big[\sum_{i=1}^{n_r}\sum_{j=1}^{n_r} n_\nu^i\ n_\nu^j \alpha_{ij}\ \delta_{th}^i\ \delta_{th}^j\Big]^{1/2} \tag{13}$$

with $\delta_{th}^i = \delta_{bias}^i + \delta_{random}^i$ will produce

$$\sigma(n_\nu^{total}) = \Big[\sum_{i=1}^{n_r} (n_\nu^i\ \delta_{random}^i)^2 + \sum_{i=1}^{n_r}\sum_{j=1}^{n_r} n_\nu^i\ n_\nu^j \alpha_{ij}\ \delta_{bias}^i\ \delta_{bias}^j\Big]^{1/2} \tag{14}$$

Taking $\delta_{bias}^i = 1/4\ \delta_{th}^i$, $\delta_{random}^i = 3/4\ \delta_{th}^i$ with biased parts maximally correlated ($\alpha_{ij} = 1$), one calculates

$$\sigma(n_\nu^{total}) = \Big[\sum_{i=1}^{n_r} (\frac{3}{4}\ n_\nu^i\ \delta_{th}^i)^2 + \sum_{i=1}^{n_r}\sum_{j=1}^{n_r} \frac{1}{16}\ n_\nu^i\ n_\nu^j\ \delta_{bias}^i\ \delta_{bias}^j\Big]^{1/2} \tag{15}$$

This would result in

$$\frac{\sigma(n_\nu^{total})}{n_\nu^{total}} = 0.484\% \tag{16}$$

of relative uncertainty in anti-neutrino rate at KamLAND due to the reactor power uncertainty as above.

5 Conclusion

If more precise informations on the systematic uncertainty of the thermal power could be provided by japanese electric companies, the relative uncertainty in anti-neutrino rate at KamLAND would go down to 0.5% or lower. That would be significant improvement compared to present 2% error.

References

[1] Private communication with Mr. Ray Foster, the Thermal Performance Engineer at Diablo Canyon Power Plant, Avila Beach, CA.

[2] http://www.awa.tohoku.ac.jp/~r_dat/reactor_data/ReactorDataInfo_5

[3] Final Results from Palo Verde neutrino oscillation experiment, PRD 64, 112001.

[4] L.H. Miler, Ph.D. thesis, Stanford University, 2000.

[5] R. Foster, Ultrasonic Flow Mesurement at Nuclear Power Plants, Flow Control Network, June 2001.

[6] US Code of Federal Regulations, 10 CFR, Part 50, Appendix K, ECCS Evaluation Model.