ChE 306: HEAT TRANSFER
FALL 2009
Homework #1
(80 points)
DUE: FRIDAY, AUGUST 28

Chapter 1 Problems

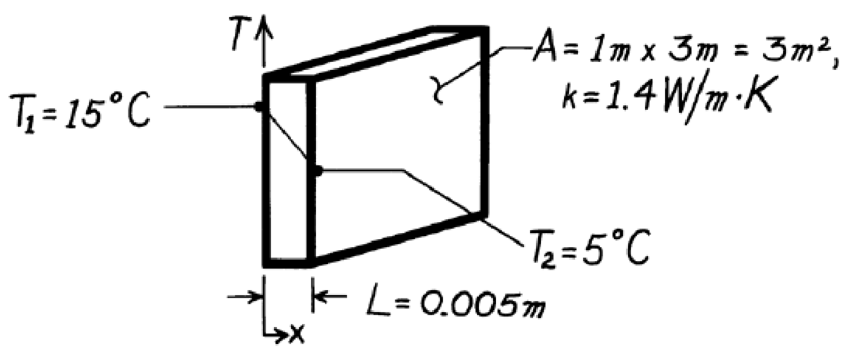
1. On a winter day, the inner surface of a glass window is 15 ^oC and the outer surface is 5 ^oC. The window is 5 mm thick, and measures 1 m by 3 m (length by width). Look up the thermal conductivity of plate glass in Appendix A3. A

A. What is the rate of heat loss through the window (in Watts)?

KNOWN: Inner and outer surface temperatures of a glass window of prescribed dimensions

FIND: Heat loss through window.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: Subject to the foregoing conditions the heat flux may be computed from Fourier's law, Eq. 1.2.

$$q''_x = k \frac{T_1 - T_2}{L}$$

$$q''_x = 1.4 \frac{W}{m \cdot K} \frac{(15\text{-}5)^\circ C}{0.005m}$$

$$q''_x = 2800 \ W/m^2.$$

Since the heat flux is uniform over the surface, the heat loss (rate) is

$$q = q''_x \times A$$

$$q = 2800 \ W/m^2 \times 3m^2$$

$$q = 8400 \ W. \qquad <$$

COMMENTS: A linear temperature distribution exists in the glass for the prescribed conditions.

Note: This positive value of q is for a system where the x direction points from hot to cold temperatures.

B. Given the same thickness and dimensions of a slab of oak, with the same temperature conditions as the glass window, what is the heat loss (in Watts)? (Use Table A3 again)

Compare your answer to (A) to see if your numbers make sense.

The only thing that changes is k. (k = 0.16 W/m-K)

$q = 0.16 * 3m^2 (15 - 5)/0.005 = 960$ W (or -960 W depending on how you defined the x direction).

Yes, wood is more insulating than glass, so heat loss is smaller.

C. Returning to the glass window, but on a summer day in Alabama, where the surface temperature of on the outside surface of the glass is 95 ^oF and the inside surface temperature is 77 ^oF, what is the rate of heat transfer into the room through the glass (in Watts)?

Converting Temperatures to Celsius, the driving force is 35 - 25 or 10 ^oC, so the dT is the same as in part (A), except that the direction is reversed. k and A are the same, so q = - 8400 W (or 8400 W going from outside to inside).

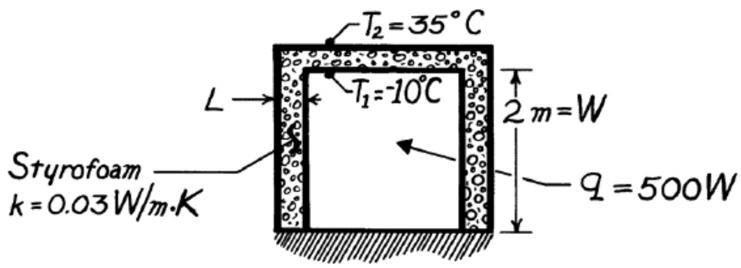
2. A home freezer that is part of a two-door refrigerator unit is kept so that the inner walls are -10 °C. The unit is placed in a carport where the outer surface is subjected to 35 °C temperatures. The compartment is a cube with 2m dimensions on each side. Assume that the bottom of the freezer is completely insulated (it's above the refrigerator).

A. What thickness of styrofoam insulation is needed to keep the heat load for the freezer less than 500 W?
B. What thickness would be required if the freezer were kept indoors (room temp of 23 °C) instead?
Note: Styrofoam has a thermal conductivity of 0.030 W/m-K.

KNOWN: Dimensions of freezer compartment. Inner and outer surface temperatures.

FIND: Thickness of styrofoam insulation needed to maintain heat load below prescribed value.

SCHEMATIC:



ASSUMPTIONS: (1) Perfectly insulated bottom, (2) One-dimensional conduction through 5 walls of area $A = 4m^2$, (3) Steady-state conditions, (4) Constant properties.

ANALYSIS: Using Fourier's law, Eq. 1.2, the heat rate is

$$q = q''\cdot A = k\,\frac{\Delta T}{L}\,A_{total}$$

Solving for L and recognizing that $A_{total} = 5\times W^2$, find

$$L = \frac{5\,k\,\Delta T\,W^2}{q}$$

$$L = \frac{5 \times 0.03\ W/m\cdot K\,[35 - (-10)]^{\circ}C\,(4m^2)}{500\ W}$$

$$L = 0.054m = 54mm.$$ <

COMMENTS: The corners will cause local departures from one-dimensional conduction and a slightly larger heat loss.

B. Keeping it inside with a 23°C outside temperature will reduce the thickness required.
$L = 5\ (0.03)\ (35\text{-}23)\ (4\ m^2)\ /\ (500\ W) = 0.0144\ m$ (or 14.4 mm)

3. Work Problem 1.13 in Incropera & DeWitt.

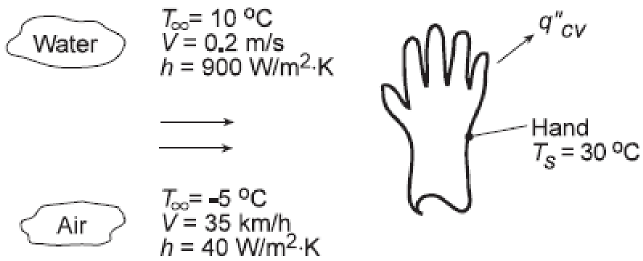
C. What would happen to q" if the velocities increased? Would any term on the right hand side of Newton's Law of Cooling change? If so, how?

PROBLEM 1.13

KNOWN: Hand experiencing convection heat transfer with moving air and water.

FIND: Determine which condition feels colder. Contrast these results with a heat loss of 30 W/m^2 under normal room conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Temperature is uniform over the hand's surface, (2) Convection coefficient is uniform over the hand, and (3) Negligible radiation exchange between hand and surroundings in the case of air flow.

ANALYSIS: The hand will feel colder for the condition which results in the larger heat loss. The heat loss can be determined from Newton's law of cooling, Eq. 1.3a, written as

$$q'' = h\left(T_s - T_\infty\right)$$

For the air stream:

$$q''_{air} = 40\,W/m^2 \cdot K\left[30-(-5)\right]K = 1,400\,W/m^2$$ <

For the water stream:

$$q''_{water} = 900\,W/m^2 \cdot K(30-10)K = 18,000\,W/m^2$$ <

COMMENTS: The heat loss for the hand in the water stream is an order of magnitude larger than when in the air stream for the given temperature and convection coefficient conditions. In contrast, the heat loss in a normal room environment is only 30 W/m^2 which is a factor of 400 times less than the loss in the air stream. In the room environment, the hand would feel comfortable; in the air and water streams, as you probably know from experience, the hand would feel uncomfortably cold since the heat loss is excessively high.

(C) If the velocity increases, we know intuitively that the rate of heat loss will increase. Even though velocity is not explicitly in the formula, you can deduce that the temperatures won't change, and neither does the surface area of your hand. Thus, the convective heat transfer coefficient, h, must increase with velocity.

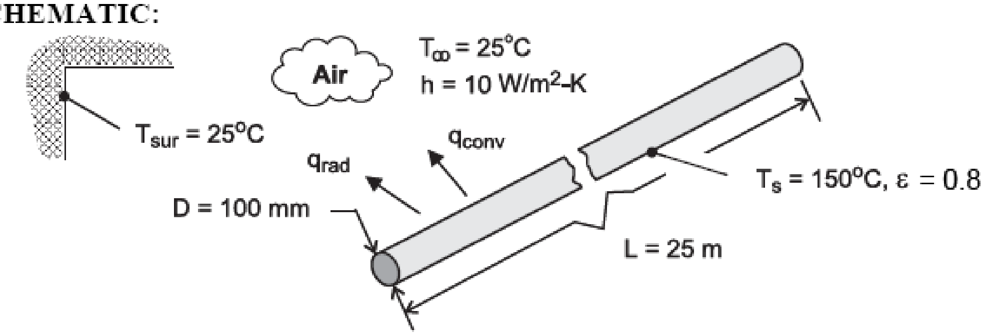
4. A steam pipe at an oil refinery passes through the air-conditioned control room, which has an air temperature of 25 °C and walls also at 25 °C. The pipe is uninsulated (metal is exposed to air) and the surface of the pipe is 150 °C. The pipe is 25 m long and has an outside diameter of approximately 4 inches (10 cm). Given that the convection coefficient for natural convection from the steam pipe to the room air is h = 10 W/m^2-K, and that the surface emissivity, ε, of 0.8:

A. Determine the rate of heat loss (q) from the pipe due to convection.
B. Determine the rate of heat loss (q) from the pipe due to radiation.
C. What is the total rate of heat loss from the pipe?
D. What would your answers to (A) and (B) be if the pipe were made of a different material with emissivity of 0.7?

KNOWN: Length, diameter, surface temperature and emissivity of steam line. Temperature and convection coefficient associated with ambient air. Efficiency and fuel cost for gas fired furnace.

FIND: (a) Rate of heat loss, (b) Annual cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steam line operates continuously throughout year, (2) Net radiation transfer is between small surface (steam line) and large enclosure (plant walls).

ANALYSIS: (a) From Eqs. (1.3a) and (1.7), the heat loss is

$$q = q_{conv} + q_{rad} = A\left[h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

where $A = \pi DL = \pi\left(0.1\text{m} \times 25\text{m}\right) = 7.85\text{m}^2$.

Hence,

$$q = 7.85\text{m}^2\left[10\ \text{W/m}^2\cdot\text{K}\left(150-25\right)\text{K} + 0.8\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(423^4 - 298^4\right)\text{K}^4\right]$$

$$q = 7.85\text{m}^2\left(1{,}250 + 1{,}095\right)\text{W/m}^2 = \left(9813 + 8592\right)\text{W} = 18{,}405\ \text{W}$$ <

From above (a): q_{conv} = 9813 W,(b) q_{rad} = 8592 W, (c) 18,405 W

(d) If the emissivity changes, the heat loss to convection would not change (still 9813 W)
qrad = 0.7 * 5.67 * 10^{-8} * (423^4 - 298^4) = 7518 W.

Chapter 2 Problems

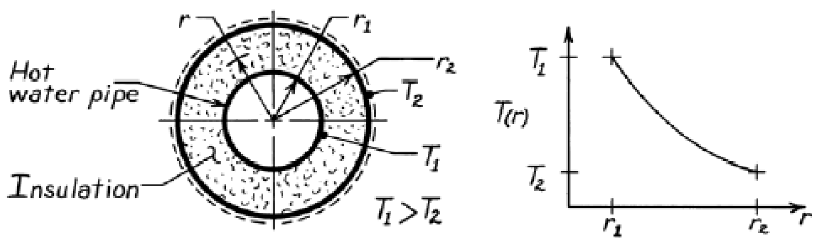
5. Work Problem 2.2 in Incropera & DeWitt.

PROBLEM 2.2

KNOWN: Hot water pipe covered with thick layer of insulation.

FIND: Sketch temperature distribution and give brief explanation to justify shape.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (radial) conduction, (3) No internal heat generation, (4) Insulation has uniform properties independent of temperature and position.

ANALYSIS: Fourier's law, Eq. 2.1, for this one-dimensional (cylindrical) radial system has the form

$$q_r = -kA_r \frac{dT}{dr} = -k\left(2\pi r\ell\right)\frac{dT}{dr}$$

where $A_r = 2\pi r\ell$ and ℓ is the axial length of the pipe-insulation system. Recognize that for steady-state conditions with no internal heat generation, an energy balance on the system requires $\dot{E}_{in} = \dot{E}_{out}$ since $\dot{E}_g = \dot{E}_{st} = 0$. Hence

$$q_r = \text{Constant.}$$

That is, q_r is independent of radius (r). Since the thermal conductivity is also constant, it follows that

$$r\left[\frac{dT}{dr}\right] = \text{Constant.}$$

This relation requires that the product of the radial temperature gradient, dT/dr, and the radius, r, remains constant throughout the insulation. For our situation, the temperature distribution must appear as shown in the sketch.

COMMENTS: (1) Note that, while q_r is a constant and independent of r, q''_r is not a constant. How does $q''_r(r)$ vary with r? (2) Recognize that the radial temperature gradient, dT/dr, decreases with increasing radius.

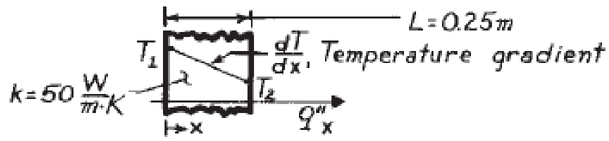
6. Work Problem 2.8 in Incropera & DeWitt.

PROBLEM 2.8

KNOWN: One-dimensional system with prescribed thermal conductivity and thickness.

FIND: Unknowns for various temperature conditions and sketch distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No internal heat generation, (4) Constant properties.

ANALYSIS: The rate equation and temperature gradient for this system are

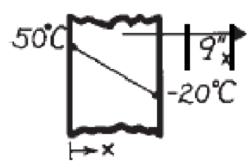
$$q''_x = -k\frac{dT}{dx} \qquad \text{and} \qquad \frac{dT}{dx} = \frac{T_2 - T_1}{L}. \qquad (1,2)$$

Using Eqs. (1) and (2), the unknown quantities for each case can be determined.

(a)

$$\frac{dT}{dx} = \frac{(-20-50)\,\text{K}}{0.25\text{m}} = -280\ \text{K/m}$$

$$q''_x = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[-280\frac{\text{K}}{\text{m}}\right] = 14.0\ \text{kW/m}^2.$$



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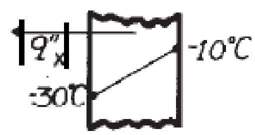
(b)

$$\frac{dT}{dx} = \frac{(-10-(-30))\,\text{K}}{0.25\text{m}} = 80\ \text{K/m}$$

$$q''_x = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[80\frac{\text{K}}{\text{m}}\right] = -4.0\ \text{kW/m}^2.$$



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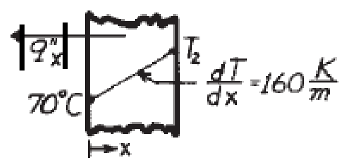
(c)

$$q''_x = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[160\frac{\text{K}}{\text{m}}\right] = -8.0\ \text{kW/m}^2$$

$$T_2 = L\cdot\frac{dT}{dx} + T_1 = 0.25\text{m} \times \left[160\frac{\text{K}}{\text{m}}\right] + 70^\circ\text{C}.$$

$$T_2 = 110^\circ\text{C}.$$



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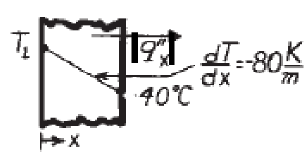
(d)

$$q''_x = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[-80\frac{\text{K}}{\text{m}}\right] = 4.0\ \text{kW/m}^2$$

$$T_1 = T_2 - L\cdot\frac{dT}{dx} = 40^\circ\text{C} - 0.25\text{m}\left[-80\frac{\text{K}}{\text{m}}\right]$$

$$T_1 = 60^\circ\text{C}.$$



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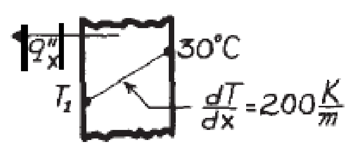
(e)

$$q''_x = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[200\frac{\text{K}}{\text{m}}\right] = -10.0\ \text{kW/m}^2$$

$$T_1 = T_2 - L\cdot\frac{dT}{dx} = 30^\circ\text{C} - 0.25\text{m}\left[200\frac{\text{K}}{\text{m}}\right] = -20^\circ\text{C}.$$



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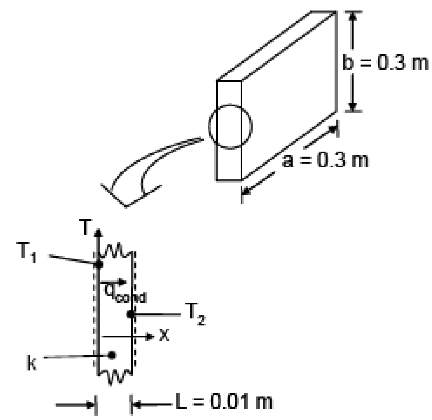
7. Work Problem 2.14 in Incropera & DeWitt.

PROBLEM 2.14

KNOWN: Dimensions of and temperature difference across an aircraft window. Window materials and cost of energy.

FIND: Heat loss through one window and cost of heating for 180 windows on 8-hour trip.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the x-direction, (3) Constant properties.

PROPERTIES: Table A.3, soda lime glass (300 K): k_{gl} = 1.4 W/m·K.

ANALYSIS: From Eq. 2.1,

$$q_x = -kA\frac{dT}{dx} = k\,a\,b\frac{(T_1 - T_2)}{L}$$

For glass,

$$q_{x,g} = 1.4\,\frac{W}{m\cdot K} \times 0.3\text{ m} \times 0.3\text{ m} \times \left[\frac{80°C}{0.01m}\right] = 1010\text{ W}$$ <

The cost associated with heat loss through N windows at a rate of R = \$1/kW·h over a t = 8 h flight time is

$$C_g = Nq_{x,g}Rt = 130 \times 1010\text{ W} \times 1\,\frac{\$}{kW\cdot h} \times 8\text{ h} \times \frac{1kW}{1000W} = \$1050$$ <

Repeating the calculation for the polycarbonate yields

$$q_{x,p} = 151\text{ W},\ C_p = \$157$$ <

while for aerogel,

$$q_{x,a} = 10.1\text{ W},\ C_a = \$10$$ <

COMMENT: Polycarbonate provides significant savings relative to glass. It is also lighter (ρ_p = 1200 kg/m^3) relative to glass (ρ_g = 2500 kg/m^3). The aerogel offers the best thermal performance and is very light (ρ_a = 2 kg/m^3) but would be relatively expensive.

8. Work Problem 2.16 in Incropera & DeWitt.
Determine the required thermal conductivity of the manufacturer's insulation, and use Appendix A3 (p 937-940) to determine what material the insulation might be made of.

PROBLEM 2.16

KNOWN: Different thicknesses of three materials: rock, 18 ft; wood, 15 in; and fiberglass insulation, 6 in.

FIND: The insulating quality of the materials as measured by the R-value.

PROPERTIES: *Table A-3* (300K):

Material	Thermal conductivity, W/m·K
Limestone	2.15
Softwood	0.12
Blanket (glass, fiber 10 kg/m^3)	0.048

ANALYSIS: The R-value, a quantity commonly used in the construction industry and building technology, is defined as

$$R \equiv \frac{L(\text{in})}{k\left(\text{Btu}\cdot\text{in/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)}.$$

The R-value can be interpreted as the thermal resistance of a 1 ft^2 cross section of the material. Using the conversion factor for thermal conductivity between the SI and English systems, the R-values are:

Rock, Limestone, 18 ft:

$$R=\frac{18\text{ ft}\times 12\dfrac{\text{in}}{\text{ft}}}{2.15\dfrac{\text{W}}{\text{m}\cdot\text{K}}\times 0.5778\dfrac{\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W/m}\cdot\text{K}}\times 12\dfrac{\text{in}}{\text{ft}}}=14.5\left(\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1}$$ <

Wood, Softwood, 15 in:

$$R=\frac{15\text{ in}}{0.12\dfrac{\text{W}}{\text{m}\cdot\text{K}}\times 0.5778\dfrac{\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W/m}\cdot\text{K}}\times 12\dfrac{\text{in}}{\text{ft}}}=18\left(\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1}$$ <

Insulation, Blanket, 6 in:

$$R=\frac{6\text{ in}}{0.048\dfrac{\text{W}}{\text{m}\cdot\text{K}}\times 0.5778\dfrac{\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W/m}\cdot\text{K}}\times 12\dfrac{\text{in}}{\text{ft}}}=18\left(\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1}$$ <

COMMENTS: The R-value of 19 given in the advertisement is reasonable.

For an R value of 19 and a thickness of 6 inches,
k = 6 in / {(19in/Btu/ft^{o}F) x 0.5778 x 12 in/ft} = 0.0455 W/m-K
this is similar to a blanket, or clay tile, or some plastering materials, or cork.