

Chapter 6 – Circular Motion and Other Applications of Newton’s Laws

Uniform Circular Motion

For an object moving in a circle with constant speed, the acceleration is directed toward the center of the circle and has magnitude

$$a_c = \frac{v^2}{r}$$

Thus, according to Newton’s 2nd law the net force is given by

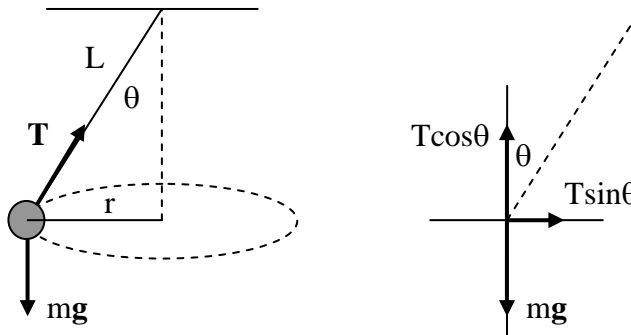
$$\Sigma \mathbf{F} = m a_c = m \frac{v^2}{r}$$

Example: A ball attached to the end of a string moves in a circular path at constant speed on a frictionless, horizontal table, with one end of the string fixed at the center of the circle. The mass of the ball is 0.5 kg, the radius of the circle is 0.8 m, and the speed of the ball is 10 m/s. What is the tension in the string?

Since the normal force exerted by the table on the ball and the weight of the ball cancel out, the net force is just the tension in the string. Thus,

$$\Sigma F = T = m \frac{v^2}{r} = (0.5 \text{ kg}) \frac{(10 \text{ m/s})^2}{0.8 \text{ m}} = \underline{62.5 \text{ N}}$$

Example: If a mass hanging from a string swings in a circle, then the string sweeps out a cone. This system is called a *conical pendulum*. (If it swings back and forth in a vertical plane, then it is a *simple pendulum*.) What is the relationship between the speed of the mass, the length of the string, and the angle that the string makes with the vertical in order to have a conical pendulum?



$$\Sigma F_x = ma_x, \text{ or}$$

$$T \sin \theta = m \frac{v^2}{r} \quad (1)$$

$$\Sigma F_y = T \cos \theta - mg = 0, \text{ or}$$

$$T \cos \theta = mg \quad (2)$$

Combining eqs. (1) and (2), we get: $\tan \theta = \frac{v^2}{rg}$

As a specific example, if the length of the pendulum is 1 m, what speed would be required to have $\theta = 30^\circ$? Solving the above equation for v and noting that $r = L \sin \theta$, we have

$$v = \sqrt{rg \tan \theta} = \sqrt{Lg \sin \theta \tan \theta} = \sqrt{(1m)(9.8m/s^2) \sin(30^\circ) \tan(30^\circ)} = \underline{1.68m}$$

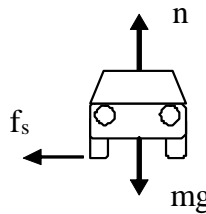
Example: How fast can a car go around a circular, flat track at constant speed without skidding?

$$\Sigma F_x = f_s = ma_c = m \frac{v^2}{r}$$

$$\Sigma F_y = n - mg = 0 \Rightarrow n = mg$$

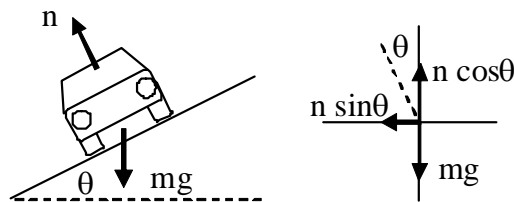
$$f_s = \mu_s n = \mu_s mg, \text{ or}$$

$$\frac{v^2}{r} = \mu_s g, \quad v = \sqrt{\mu_s gr}$$



For example, if $r = 200$ m and $\mu_s = 0.4$, then $v_{\max} = 28$ m/s (= 63 mph).

Example: What is the optimum angle of bank of a circular track? The optimum angle would be such that no frictional force acts to cause the car to skid up the bank or slip down the bank.



$$\Sigma F_x = n \sin \theta = m \frac{v^2}{r}$$

$$\Sigma F_y = n \cos \theta - mg = 0 \Rightarrow n \cos \theta = mg$$

From the above two equations we get

$$\tan \theta = \frac{v^2}{gr}$$

So, if $v = 28 \text{ m/s}$ and $r = 200 \text{ m}$ (the numbers for the flat track), then

$$\tan \theta = \frac{(28 \text{ m/s})^2}{(9.8 \text{ m/s}^2)(200 \text{ m})} = 0.4 \Rightarrow \theta = 21.8^\circ$$

Example: Merry-go-round.

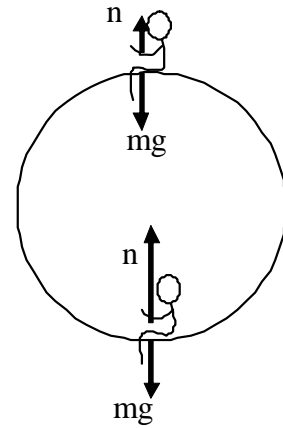
The net force and the acceleration are always toward the center of the circle.

Bottom:

$$n - mg = m \frac{v^2}{r}, \quad n = m \left(g + \frac{v^2}{r} \right) = mg \left(1 + \frac{v^2}{rg} \right)$$

Top:

$$mg - n = m \frac{v^2}{r}, \quad n = m \left(g - \frac{v^2}{r} \right) = mg \left(1 - \frac{v^2}{rg} \right)$$



The 'effective' weight of the person on the merry-go-round is the normal force (the force exerted by the seat on the person). Thus, at the bottom $W_{\text{eff}} > mg$ and at the top $W_{\text{eff}} < mg$.

For example, if $r = 10 \text{ m}$ and the time for a revolution = 30 s , then

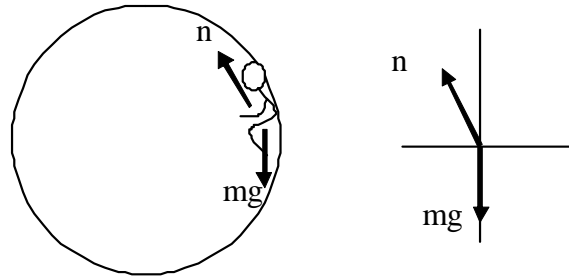
$$v = \frac{2\pi r}{T} = \frac{2\pi (10 \text{ m})}{30 \text{ s}} = 2.09 \text{ m/s}$$

$$\frac{v^2}{rg} = \frac{(2.09 \text{ m/s})^2}{(10 \text{ m})(9.8 \text{ m/s}^2)} = 0.045$$

Thus, a person's effective weight would be increased or decreased by 4.5%.

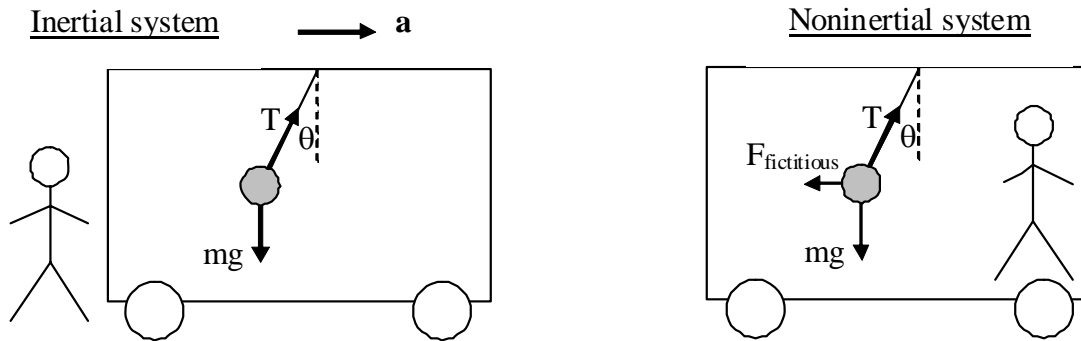
What would be the effective weight at the side of the ferris wheel?

In this case, the seat would tilt so that the normal force (and the effective weight) would be at an angle. You should be able to show that



$$n = mg \sqrt{1 + \frac{v^2}{rg}}$$

Example: A ball on the end of a string hangs from the ceiling of a linearly accelerating car.



To an observer standing at rest outside the car, the ball is accelerating and Newton's 2nd law gives

$$\begin{aligned} \Sigma F_x &= T \sin \theta = ma \\ \Sigma F_y &= T \cos \theta - mg = 0 \end{aligned}$$

This person is said to be in an 'inertial' coordinate system.

A person inside the car sees the ball at rest and in equilibrium. Thus, there appears to be a fictitious force acting on the ball to make it hang at an angle. This fictitious force has magnitude $F_{fic} = ma$ and is directed to the left. He could also conclude that gravity appears larger with $g_{eff} = \sqrt{g^2 + a^2}$ and with the 'down' direction inclined at an angle θ . Since Newton's laws don't hold in the accelerating system without invoking a fictitious force, this system is called a 'noninertial' system.