Module #21

Grain Size Hardening

(“Hall-Petch Relationship”)

Grain boundary strengthening (1)

- Grain boundaries also impede dislocation motion. Thus, they also contribute to strengthening.

- The *magnitude of* the observed *strengthening depends upon* the *structure of the grain boundaries* and the degree of *misorientation between grains*.

- Several models have been developed to describe grain boundary strengthening. Interestingly, nearly all of them can be reduced to the Hall-Petch relationship as originally proposed by Hall (1951) and Petch (1953).

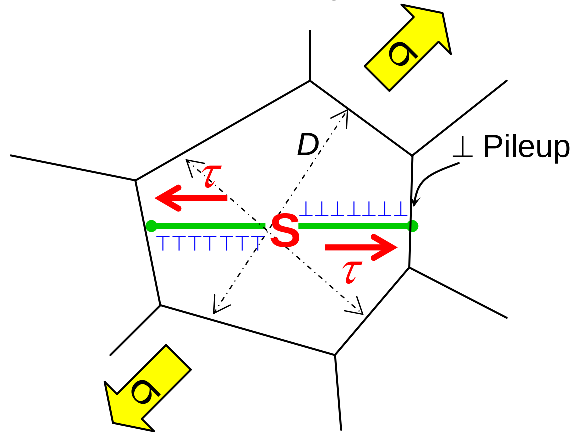
  – E.O. Hall, *Proceedings of the Physical Society B*, 64 (1951) p. 747.

  – N.J. Petch, “The cleavage strength of polycrystals,” *J. Iron Steel Inst.*, 174 (1953), p. 25.

- One such model is provided on the next view graph.

Grain boundary strengthening (2)

- Consider a grain that contains a single dislocation source in its center.



- The original Petch model is based on the concept that dislocations emitted from point sources within individual grains (perhaps from Frank-Read sources) will encounter a *lattice friction stress* τ_o (i.e., a Peierls stress) as they move on a slip plane towards a grain boundary.
- This lattice friction stress opposes the *applied shear stress* τ_a.
- The *effective shear stress* τ_{eff} that contributes to plastic deformation (i.e., the stress to make a dislocation move) is then given by:

$$\tau_{eff} = \tau_a - \tau_o$$

- However, since dislocation motion is impeded by grain boundaries, dislocations will pile up until the stress is large enough for them to break through the grain boundary.

Grain boundary strengthening (3)

$$\tau_{eff} = \tau_a - \tau_o$$

- In this model, the shear stress at the grain boundary is given by:

$$\tau_{gb} = \tau_{eff}\sqrt{\frac{D}{4r}} = (\tau_a - \tau_o)\sqrt{\frac{D}{4r}}$$

$D =$ grain size

$r =$ distance from source

- For explanation only, if we assume that bulk yielding occurs at a critical value of τ_{gb}, we can rearrange the preceding equation in terms of the applied shear stress.

$$\tau_a = \tau_o + \tau_{gb}\sqrt{\frac{4r}{D}}$$

- τ_{gb} and *r* are essentially constant, which reduces this equation

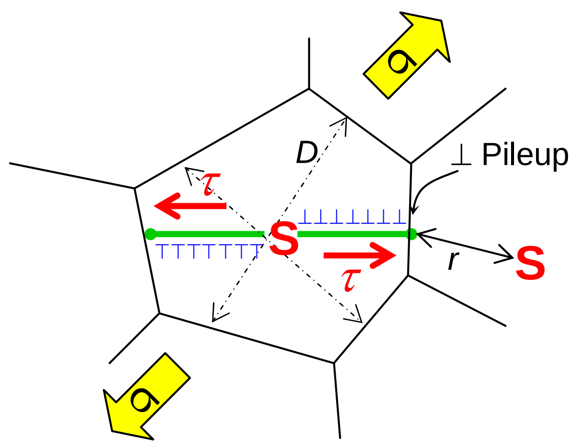
$$\tau_y = \tau_o + k_y' D^{-1/2}$$ or $$\sigma_y = \sigma_o + k_y D^{-1/2}$$

This form of the HP equation is derived when we incorporate the Taylor factor, thus turning shear stress into a normal stress.

There are other models for GB strengthening
(I've outlined a few on the next 4 pages)

Grain boundary strengthening (4)

- Cottrell (Trans. *TMS-AIME*, 212, 1958, p. 192) modified the original Hall-Petch model.
- Why? He recognized that *it is virtually impossible for dislocations to burst through grain boundaries as described by Hall & Petch*.



- Assumed that stress concentration that produced a pileup in one grain activated a dislocation source in an adjacent grain.
- The maximum shear stress at a distance r ahead of the boundary is given by:

$$\tau = (\tau_a - \tau_o)\sqrt{\frac{D}{4r}}$$

where τ is the stress required to activate the source in the adjacent grain (i.e., stress required to initiate dislocation motion); τ_a is the applied shear stress at which the source becomes active; and τ_o is the Peierls stress. In this model, r < D/2.

- $(D/4r)^{1/2}$ represents the stress concentration arising from the pileup. It increases as the number of dislocations increases (thus it ↑ as *D* ↑).

Grain boundary strengthening (5)

$$\tau = (\tau_a - \tau_o)\sqrt{\frac{D}{4r}}$$

- Assuming that $\tau_a = \tau_y$, this equation can now be rewritten as:

$$\tau_a = \tau_o + \tau\sqrt{\frac{4r}{D}} = \boxed{\tau_y = \tau_o + k_y' D^{-1/2}}$$

or

$$\boxed{\sigma_y = \sigma_o + k_y D^{-1/2}}$$

- The Hall-Petch and Cottrell models have physical appeal. However, very few investigators have observed dislocation pileups at boundaries.

Grain boundary strengthening (6)

- The Hall-Petch and Cottrell models have physical appeal. However, very few investigators have observed dislocation pileups at boundaries.
- Li (*Trans. TMS-AIME*, 227, 1963, p. 239) considered that grain size effects were caused by dislocation emission from grain boundary ledges.
- In this model, the ability of a grain boundary to emit dislocation is characterized by new parameter μ.

$$\mu = \frac{\text{total length of } \perp \text{ line emitted}}{\text{unit area of grain boundary}}$$

- The parameter μ is also related to the dislocation density at yielding, $\rho_\perp$, by the relation:

$$\rho_\perp = \frac{3\mu}{D}$$

- Recall from our lectures on work hardening the following:

$$\tau = \tau_o + \alpha G b \sqrt{\rho_\perp}$$

- Combining terms, we obtain:

$$\tau_y = \tau_o + \alpha G b \sqrt{\frac{3\mu}{D}} = \tau_o + k_y' D^{-1/2} \text{ or } \boxed{\sigma_y = \sigma_o + k_y D^{-1/2}}$$

Grain boundary strengthening (7)

$$\tau_y = \tau_o + \alpha G b \sqrt{\frac{3\mu}{D}} = \tau_o + k'_y D^{-1/2} \text{ or } \boxed{\sigma_y = \sigma_o + k_y D^{-1/2}}$$

A FEW IMPORTANT THINGS TO CONSIDER:

- k_y increases when the Schmid factor *m* increases.
- Higher values of k_y correlate with increasing strength.
- In metals it is “usually” necessary to produce grains that are smaller than 5 μm in diameter to gain appreciable strengthening.
- However, there are some notable exceptions…

Ex.: Nanocrystalline materials

- These are materials with grain sizes that are less than 100 nm (generally with GS near 10 nm).
- They can be viewed as composites consisting of small dislocation-free crystals in an amorphous matrix. Grain boundaries are on the order of 5 nm in width.
- Due to their small grain sizes, high applied stresses will be required to activate sources and to build up suitable stress concentrations at grain boundaries according to the models presented above.
- Also, the amorphous grain boundaries will be barriers to slip transfer. This is worth consideration in greater detail.

Grain boundary strengthening (8)

$$\sigma_y = \sigma_o + k_y D^{-1/2} \quad \text{or} \quad \Delta\sigma_{gb} = k_y D^{-1/2}$$

MORE IMPORTANT THINGS TO CONSIDER:

- The Hall-Petch relationship has been reported in a large number of crystalline materials.
- The degree to which grain boundary (i.e., grain size) strengthening can be effectively used depends on the material's Hall-Petch coefficient and the degree of grain-size refinement possible in the material.
- Example:
  - Engineered ceramics generally have finer grain sizes than metals and are thus much stronger. Of course this is also related to their complex crystal structures.
  - Fine grained ceramics are stronger and more fracture resistant than their coarse grained counterparts.
  - Nanocrystals?

Combination of Strengthening Mechanisms
(up to this point)

MORE IMPORTANT THINGS TO CONSIDER:

- For a single phase polycrystalline alloy with a dislocation density of $\rho_\perp$, the strength increase due to work hardening and grain size hardening can be crudely estimated by superimposing some of the terms that we have derived thus far.

$$\Delta\sigma = \Delta\sigma_\perp + \Delta\sigma_{gb} + \Delta\sigma'_\perp$$

$$= k_\perp\sqrt{\rho_\perp} + \frac{k_y}{\sqrt{D}} + \frac{k'_\perp}{\sqrt{s}}$$

- This of course neglects the possibility of other strengthening mechanisms and any potential interactions between them. Keep in mind, other things do occur which significantly complicate our ability to develop generally applicable descriptions of strengthening.