FIXED IMPULSE TRANSFERS

Introduction

Many of the satellites placed into orbit from the Space Shuttle utilize solid rocket boosters to attain their final orbit. These types of engines usually do not incorporate a way to extinguish the burning propellant until all of it is spent. This poses a problem for mission designers since the ΔV requirements for coplanar and non-coplanar orbit changes are fixed. For example, suppose that a communications satellite is to be placed in orbit after being released from the Space Shuttle in a low Earth orbit, often abbreviated LEO. If the designer wishes to place the satellite into its final orbit, which is much higher than the orbit of the Space Shuttle, then a two-stage booster rocket will be needed, the first stage supplying ΔV_1 and the second stage supplying ΔV_2. If these engines are solid rocket boosters, then the ΔV's are fixed, depending on the mass of the payload (see **Fig. 1**).

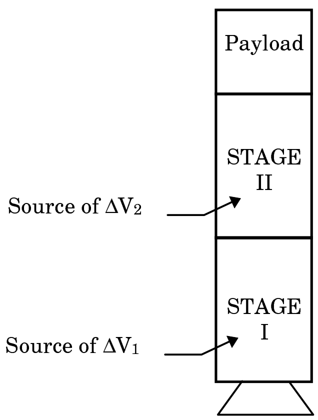

Fig. 1. Simplified Diagram Showing the Minimum Configuration Necessary to Place the Payload into the Final Orbit.

The ΔV's are calculated from the rocket equation, which is

$$\Delta V = g_0 I_{sp} \ln\left(\frac{W_{initial}}{W_{final}}\right)$$

where

g_0 is the acceleration due to gravity at the earth's surface,

I_{sp} is the specific impulse of the engine (a measure of the thrust of the engine per unit weight of propellant flowing out of the engine per second, resulting in units of N·s/N, or just s),

$W_{initial}$ is the weight of the rocket (and not just the engine) at the beginning of the burn,

W_{final} is the weight of the rocket (and not just the engine) at the end of the burn.

If the designer wishes to use a Hohmann transfer to place the satellite into its final orbit, then the ΔV values are fixed. However, the ΔV values available from the engines may not (and most likely *will not*) match the mission requirements. Therefore, the designer has basically three options:

1. Remove an appropriate amount of propellant
2. Add weight to the payload
3. Modify the trajectory

While removing propellant may sound reasonable, this can be an expensive operation, and may require re-certifying the engines for operation. Do you think that administrators will go for that?

Adding weight to the payload is an option, but seems wasteful. This is similar to adding weight to a Porsche 911 Turbo to slow it down, an unthinkable event!

Since the two boosters may have to carry a variety of payloads, it seems best to modify the trajectory to fully use the available ΔV's. Since the problem will *always* be excess energy (if there is too little energy, then the final orbit is not accessible), a combination size and plane change can be performed at both burns, even if the purpose of the second plane change is to return the orbit to its original inclination! This may seem wasteful, but the cost of wasting some propellant is less than redesigning the solid rocket booster, which would most likely be a multi-year endeavor.

Therefore, the problem reduces to making the ΔV's required for the orbital maneuvers match the capabilities of the solid rocket boosters, which is the opposite of what we have seen previously in this class. The solution to this type of problem is best learned by example.

Example of a Fixed Impulse Transfer

(This example is from Chobotov's book. See *References* at the end of this paper for more information.) The Space Shuttle delivers a satellite to an orbit 278 km above the surface of the earth, but the altitude of the target orbit is 20187 km with an inclination of 55^0. **Table 1** lists the properties of the rocket which will take the satellite to its final orbit. If a Hohmann transfer will be used, design the transfer orbit which matches the ΔV's of the engines.

Table 1. Properties of the Booster Stages and Payload.

section		**mass (kg)**
payload		907
Stage II (upper)	structure	91
	propellant	898
Stage I (lower)	structure	181
	propellant	2174

I_{sp} = 300 s for both engines; g_o = 9.8066 m/s^2.

From the rocket equation, $\Delta V_1 = 2107$ m/s and $\Delta V_2 = 1888$ m/s. (The student should verify this.) No information was given in the problem statement about the inclination of the Space Shuttle's orbit, so we will solve this for the general case. We need to perform plane changes that will use all of the propellant in the rocket engines. This is accomplished by examining the plane change geometry at the two burns, illustrated in **Fig. 2**.

Let's call the unknown plane change angles α_1 and α_2., the velocity of the initial and final circular orbits as $V_{c,1}$ and $V_{c,2}$ respectively, and the velocity at perigee and apogee of the transfer orbit V_P and V_A respectively. Since we know the ΔV's, we must find the plane changes α_1 and α_2 that will expend all the propellant but still result in a Hohmann transfer.

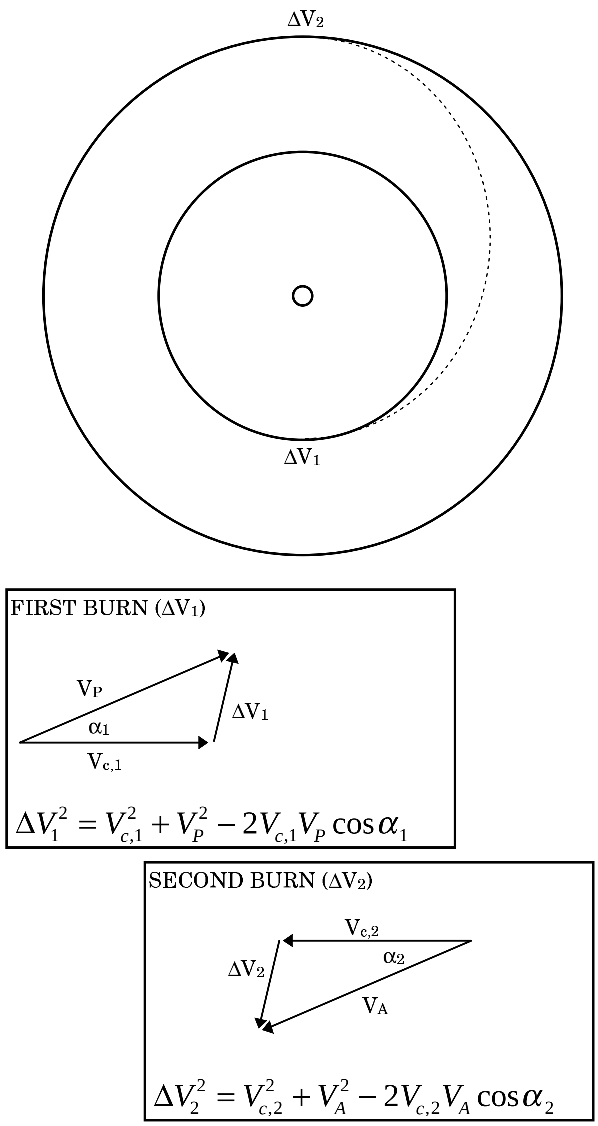

Fig. 2. Hohmann Transfer with Split Plane Change. The initial and final circular orbits are not necessarily in the same plane, although they appear to be in this sketch.

First, the velocities for the circular orbits are calculated:

$V_{c,1} = 7739$ m/s
$V_{c,2} = 3874$ m/s.

Next, the semimajor axis of the transfer orbit is found:

$a_{transfer} = 16611$ km.

This allows calculation of the perigee and apogee velocities of the transfer ellipse:

$V_P = 9787$ m/s
$V_A = 2452$ m/s.

Solving the equations in **Fig. 2** for the α values,

$\alpha_1 = 3.25$ degrees
$\alpha_2 = 23.25$ degrees.

Therefore, if we change inclination by 3.25 degrees during the first burn and by 23.25 degrees during the second burn, we can perform a Hohmann transfer *and* expend all of the propellant. Note that the direction of the plane changes are not specified, only their magnitudes. Thus the first burn could be a decrease in inclination and the second and increase, resulting in a an overall inclination change of $\theta = 20$ degrees. Or, both could be in the same direction, resulting in an inclination change of $\theta = 26.5$ degrees. This means that if the inclination of the final orbit needs to be 55 degrees, the Space Shuttle needs to be in a orbit of inclination 35 (or 75) degrees or 28.5 (or 81.5) degrees.

However, an inclination change results in a change in the longitude of the ascending node (Ω) if the burn does not occur at the equator. We can use this to our advantage. In general,

$$\cos\theta = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos\Delta\Omega$$

Since i_2 and θ are fixed by the mission requirements, we can let i_1 vary from 28.5 degrees to 81.5 degrees and solve for $\Delta\Omega$ and plot this. This plot is the outer "circle" in **Fig. 3**. This represents all possible Shuttle inclinations which will allow the proper plane change to take place and expend all of the fuel in the rockets, with both plane changes being in the same direction (both either increasing or decreasing the inclination). The inner circle represents the same concept except that the two inclination changes are in opposite directions.

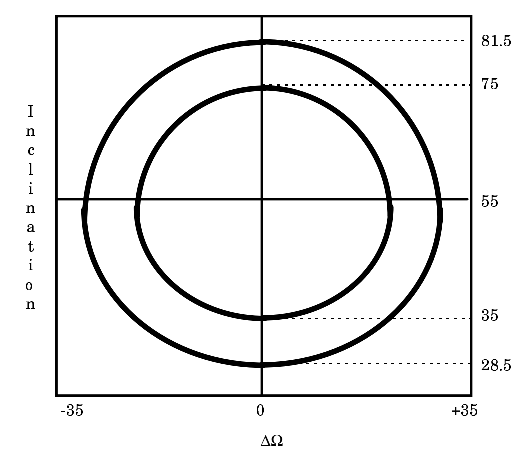

Fig. 3. Relationship between Hohmann Transfers for Fixed Impulse Plane Change.

References

Chobotov, V. A., *Orbital Mechanics*. AIAA, Washington, DC: 1991, pp 139-46.