# Slonczewski spin-torque as negative damping: Fokker-Planck computation of switching rates

D. M. Apalkov* and P. B. Visscher†
*MINT Center and Department of Physics and Astronomy*
*University of Alabama, Tuscaloosa AL 35487-0324*
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In this paper we study the application of the Fokker-Planck (FP) equation to the dynamics of a thin-film element subject to a current-induced spin torque. We use a recently-derived FP equation for the energy distribution to find an exact non-equilibrium steady state of the spin-torque system. In the present paper we verify this distribution by Landau-Lifshitz simulation. We find that the exact distribution can be expressed in terms of an energy-dependent effective temperature $T_{\text{eff}}(E)$, which turns out to be roughly constant, thus providing a precise foundation for heuristic "effective spin temperature" viewpoints.

Recent experimental evidence suggests that the phenomenon of spin-torque switching, which has possible applications to information storage, can be thought of as arising from spin-wave heating[1]. In this paper we provide a consistent theoretical basis for this previously heuristic viewpoint, using a Fokker-Planck equation for the magnetic system in the presence of a spin-polarized current. This method allows quantitative calculation of switching rates on experimental time scales not accessible to micromagnetic simulation, as well as calculation of low-frequency noise.

In spin-torque switching, a large current passing between a pinned ferromagnetic layer and a free FM layer switches the free layer[2]. The spin-wave heating viewpoint is that the spin-torque effect increases the Arrhenius factor $\exp(-E/kT)$ in the switching rate, not by lowering the barrier[3,4] $E$, but by raising the effective spin temperature $T$. To calculate this effect quantitatively, we have extended Kramers' 1940 treatment[5] of reaction rates, deriving and solving a Fokker-Planck equation for the energy distribution including a current-induced spin torque of the Slonczewski type[6,7]. In a steady state situation such as a telegraph noise experiment, we can obtain an essentially exact numerical solution of the Fokker-Planck equation. The result is a Boltzmann-like distribution with an energy-dependent effective temperature $T_{\text{eff}}(E) = T\alpha/\left[\alpha - \eta(E)J\right]$ (Eq. 8), which can be larger than the actual temperature (related to the Langevin noise through a fluctuation-dissipation theorem (Eq. 10) for one sign of the current $J$. It can be seen that the current contributes an effective negative contribution to the damping. This does not mean that the Slonczewski torque acts like Landau-Lifshitz damping at every point along an orbit, but that its overall effect over an orbit is to increase or decrease the energy, so its effect on the energy distribution is similar to that of the Landau-Lifshitz damping; the coefficient $\eta(E)$ is a ratio of two integrals over an orbit at energy $E$, essentially the ratio of the spin-torque energy loss to the Landau-Lifshitz energy loss.

This method can be used to calculate slow switching rates without long-time simulations[8]. The method should also be useful for calculating current-induced magnetic noise in CPP (current perpendicular to plane) spin valve read heads[9,10].

We will begin by summarizing the Fokker-Planck (FP) approach. In general, the FP equation gives the time evolution of a phase space probability density $\rho(p, q, t)$; it was first applied to chemical rate problems in 1940 by Kramers[5]. Kramers also derived a FP equation in energy giving the evolution of the energy distribution $\rho(E, t)$, in the special case in which the probability depends only on energy. He used this to study the reaction rate in the "low friction" case (corresponding in our case to low Landau-Lifshitz damping parameter $\alpha$), in which energy is nearly conserved and the system diffuses slowly in energy. When Brown generalized the FP equation to a magnetic system, he did this in terms of a distribution of magnetization $\rho(\mathbf{M}, t)$, (or of angles in spherical coordinates), rather than a distribution in energy $\rho(E, t)$. A key result of our recent work[8] is that the "FP equation in energy" is not only useful for the low friction case, but is exact in any steady-state distribution, and can be generalized to include a spin torque. This is important because the usual Boltzmann equilibrium distribution cannot be defined in the presence of spin torque, since the latter is not conservative.

The Landau-Lifshitz (LL) equation[11] for the evolution of a uniform magnetization $\mathbf{M}(t)$ has a deterministic and a random part:

$$\dot{\mathbf{M}} \equiv \frac{d\mathbf{M}}{dt} = \dot{\mathbf{M}}_{\text{det}} + \dot{\mathbf{M}}_{\text{rand}} \tag{1}$$

The deterministic part has a conservative precession term and the dissipative Landau-Lifshitz damping, and we will include also the Slonczewski current-induced torque[7,12]:

$$\begin{aligned} \dot{\mathbf{M}}_{\text{det}} &= -\gamma\mathbf{M} \times \mathbf{H}_{\text{cons}} - \gamma\alpha M_s \hat{\mathbf{m}} \times (\hat{\mathbf{m}} \times \mathbf{H}_{\text{cons}}) - \\ &\quad \gamma J M_s \hat{\mathbf{m}} \times (\hat{\mathbf{m}} \times \hat{\mathbf{m}}_{\text{p}}) \end{aligned} \tag{2}$$

In the first (precession) term, $\gamma$ is the gyromagnetic ratio and the conservative field is defined[8] so $\mu_0\mathbf{H}_{\text{cons}}$

is the gradient of the Stoner-Wohlfarth energy density. For a thin-film element (Fig. 1) this is (in SI units)

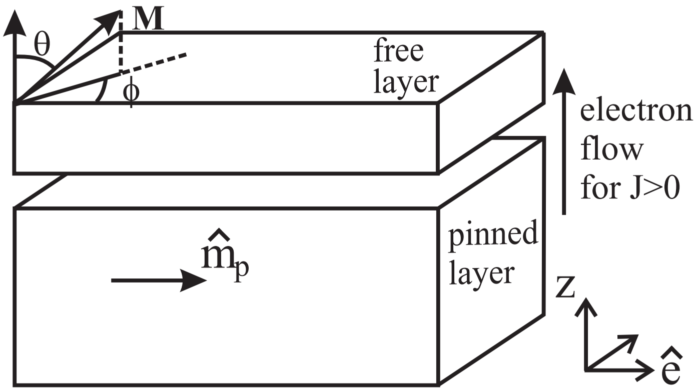


FIG. 1: Geometry of thin-film element, for the case where the magnetization $\hat{\mathbf{m}}_{\mathrm{p}}$ of the "pinned" layer is along the easy axis $\hat{\mathbf{e}}$ of the free layer.

$$E(\mathbf{M})/\mu_0 = -K(\hat{\mathbf{m}}\cdot\hat{\mathbf{e}})^2 + \frac{1}{2}M_s^2(\hat{\mathbf{m}}\cdot\hat{\mathbf{z}})^2 - \mathbf{H}_{\mathrm{ext}}\cdot\mathbf{M} \quad (3)$$

Here $\mathbf{H}_{\mathrm{ext}}$ is an external field (taken here to be zero), $K$ is the uniaxial anisotropy energy, $\hat{\mathbf{m}} \equiv \mathbf{M}/M_s$, $\hat{\mathbf{e}}$, and $\hat{\mathbf{z}}$ are unit vectors along the magnetization, easy axis, and z axis (perpendicular to the film) respectively, and $M_s$ is the saturation magnetization. In the second term of Eq.2, $\alpha$ is the dimensionless LL damping constant, assumed small so this expression is equivalent to the Gilbert formulation[11]. In the last (Slonczewski spin-torque[7,12]) term, $J$ is an empirical constant with units of magnetic field, proportional to the current density, and $\hat{\mathbf{m}}_{\mathrm{p}}$ is the magnetization direction in the pinned layer.

The directions of these torques are shown in insets to Fig. 2, which shows the contours of constant energy. The basic mechanism of spin-torque switching can be seen from Fig. 2: the Slonczewski torque pulls the magnetization out of the initial ($\phi = 0$) energy well and allows it to jump to the $\phi = \pi$ well; the Slonczewski torque generally opposes the LL damping.

The Fokker-Planck equation can be written in the form of a continuity equation[13]

$$\frac{\partial \rho(\mathbf{M},t)}{\partial t} = -\nabla\cdot\mathbf{j}(\mathbf{M},t) \quad (4)$$

where the probability current $\mathbf{j}$ along the sphere has a convective and a diffusive part:

$$\mathbf{j}(\mathbf{M},t) \equiv \rho(\mathbf{M},t)\dot{\mathbf{M}}_{\mathrm{det}}(\mathbf{M}) - D\nabla\rho(\mathbf{M},t) \quad (5)$$

(both the divergence and the gradient are two-dimensional here). This FP equation is very difficult to solve in the general case. However, in a steady state problem such as telegraph noise, the probability density depends only on energy. Even in the non-steady state

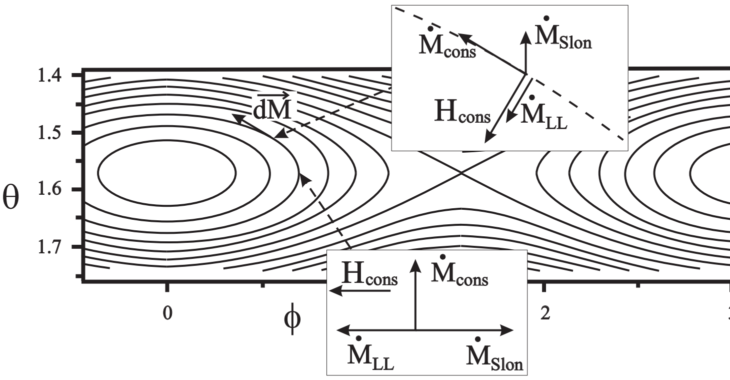


FIG. 2: Energy contours (Stoner-Wohlfarth orbits) for a thin film, plotted in terms of the coordinates $\theta$ and $\phi$ defined in Fig. 1, for the case $\mathbf{H}_{\mathrm{ext}} = 0$. The vertical scale is exaggerated for clarity. Lower inset: contributions to the rate of change of magnetization for a magnetization in the film plane. The insets show the tangent plane: magnetization points out of the paper. Upper inset: the same for an arbitrarily chosen direction of $\mathbf{M}$.

problem of escape from a metastable well, this is approximately true except near the barrier[5,13]. So it is useful to derive from Eq. 4 a FP equation in energy: some algebra[8] gives a continuity equation

$$\frac{\gamma M_s P(E)}{\mu_0}\frac{\partial \rho'(E,t)}{\partial t} = -\frac{\partial}{\partial E}j_E(E,t) \quad (6)$$

where the current $j_E(E,t)$ is the number of systems per unit time crossing a constant-energy contour. [The left hand side involves the orbital period $P(E)$ because $\rho'$ is not the probability per unit energy but per unit area on the $M$-sphere.] The current is

$$j_E(E,t) \;=\; -\gamma\alpha M_s\rho'(E,t)I^E(E) + \gamma J M_s\rho'(E,t)\hat{\mathbf{m}}_{\mathrm{p}}\cdot\mathbf{I}^M - D\frac{\partial\rho'(E,t)}{\partial E}I_i(E) \quad (7)$$

in terms of a damping integral and a magnetization integral[8]

$$I^E(E) \equiv \oint H_{\mathrm{cons}}dM \text{ and } \mathbf{I}^M(E,t) = \oint [d\mathbf{M}\times\hat{\mathbf{m}}]$$

and a diffusion term.

In steady state ($\partial\rho'/\partial t = 0$) the continuity equation (Eq. 6) gives

$$\frac{\partial \ln\rho'(E)}{\partial E} = \frac{\gamma M_s}{D}\left[-\alpha + \eta(E)J\right] \equiv -V\beta(E) \quad (8)$$

where the right hand side defines an effective inverse temperature $\beta(E)$, and $V$ is the volume of the switching element. We have also defined a quantity $\eta(E)$ with dimensions of (magnetic field)$^{-1}$ by

$$\eta(E) = \frac{\hat{\mathbf{m}}_{\mathrm{p}}\cdot\mathbf{I}^M(E)}{I^E(E)}. \quad (9)$$

Eq. 8 shows clearly that the Slonczewski torque acts like a correction to the LL damping.

Note that in the special case $J = 0$, we get the expected Boltzmann distribution with $\beta = 1/k_BT$ only if

$$D = \gamma M_s \alpha k_B T/V; \tag{10}$$

this is equivalent to the fluctuation-dissipation theorem. It allows us to eliminate the diffusivity $D$ and write the effective temperature as

$$\beta(E) = \frac{1}{k_BT}[1 - \eta(E)J/\alpha] \tag{11}$$

Integrating Eq. 8 from the bottom of the well gives

$$\rho'(E) = \rho'(E_{\min}) \exp\left[-V \int_{E_{\min}}^{E} \beta(E')dE'\right] \tag{12}$$

To the extent that the effective inverse temperature $\beta(E)$ is constant, i.e. that $\eta(E)$ is constant or $J$ is small, this is essentially a Boltzmann distribution at the effective temperature.

To check the correctness of our Fokker-Planck equation and its prediction of the energy distribution (Eq. 12), we have done LL simulation of a specific model. This is a uniformly magnetized cobalt film (similar to that used in the Cornell experiments[2]) with $M_s = 1.45 \times 10^6$ a/m ($4\pi M_s = 18.2$ kOe), $H_K \equiv 2K/(\mu_0 M_s) = 4.0 \times 10^4$ a/m (500Oe). The random field is determined by $KV/k_BT$, which we took to be 25; at $T = 300$ K, this corresponds to about $V = 2860$ nm$^3$. For these parameters, the ratio $\eta(E)$ is given in Fig.3 – note that it is very nearly constant, especially near the bottom of the well. Thus we expect the distribution (Eq. 12) to be very nearly a Boltzmann distribution with an increased energy. A Boltzmann distribution would be a straight line on Fig. 4; the prediction and the simulation are in excellent agreement, and quite close to a straight line. This agreement confirms the view that the effect of the Slonczewski spin torque is to increase the effective temperature, and that it is well-described by our generalized Fokker-Planck equation. The current parameter $J$ in the simulation was limited by the fact that switching sets in at around $J = 0.3$ kOe; simulations are planned at higher currents to test the Fokker-Planck prediction of the switching rate.

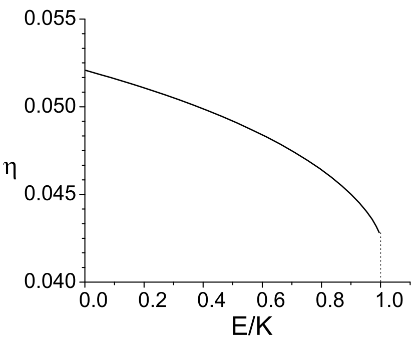


FIG. 3: The spin-torque damping coefficient $\eta(E)$ for a cobalt element. The maximum variation is about 20%, leading to nearly-Boltzmann energy distributions.

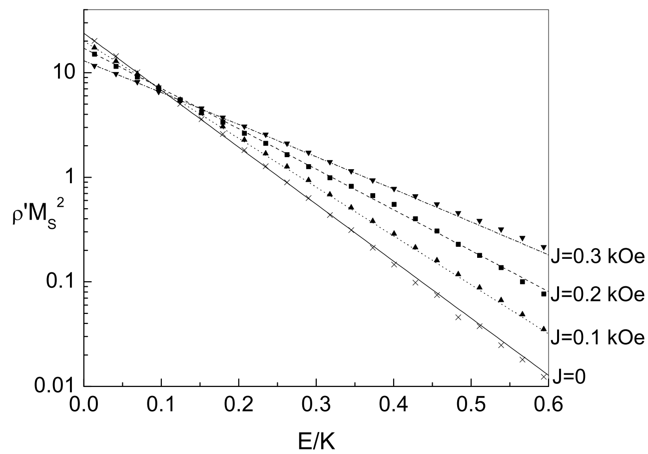


FIG. 4: The probability density $\rho'(E)$ for a cobalt element. Solid lines are the predictions of our Fokker-Planck equation, labeled by the current parameter $J = 0, 0.1, 0.2$, and 0.3 kOe. Temperature enhancement factor $(1-\eta J/\alpha)^{-1}$ is $1., 1.17, 1.42$ and 1.8, respectively. Symbols are simulation results, averaged over 60 runs, each of which was started along the easy axis and equilibrated for 34 ns before collecting data for another 68 ns.

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* Electronic address: apalk001@ua.edu
† Electronic address: visscher@ua.edu; URL:

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