MATH126
Review for Test 1

1. (a) Plot two points $P = (3, -2, 2)$ and $Q = (1, 2, 3)$ in the following coordinate system, then sketch the directed line segment PQ.

(b Write the vector **v** that is represented by PQ.

(c) Draw the vector **v** in the same picture above.

2. Let S be the point $(1, 3, -2)$.
(a) What are the coordinates of the projection of S on the xz-plane?

(b) Find the distance from S to the x-axis.

(c) Write the equation of the plane passing through S and perpendiclar to the x-axis.

3. (a) Write the equation of the sphere having center at $(1, 2, -1)$ and radius 4.
(b) What is the intersection of this sphere and the xz-plane?

4. Assume that $\mathbf{a} = 8\boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k}$; $\mathbf{b} = 3\boldsymbol{i} + 11\boldsymbol{j} - 2\boldsymbol{k}$ and $\mathbf{c} = 3\boldsymbol{i} - 4\boldsymbol{k}$
(a) Find the vector $2\mathbf{a} - 3\mathbf{b}$.
(b) Find the dot product $\mathbf{a} \cdot \mathbf{b}$.
(c) Find the component of **b** along the direction of **a**.
(d) Find the cross product $\mathbf{b} \times \mathbf{c}$.
(e) Find the volume of the parallelepiped with **a, b** and **c** as three adjacent sides.

5. Assume that **a** and **b** are given in the picture with $|\mathbf{a}| = 3$ and $|\mathbf{b}| = 2$ (**a** is in the first quadrant of the xy-plane and **b** is along the positive z-axis). Find
(a) The dot product $\mathbf{a} \cdot \mathbf{b}$.
(b) The $|\mathbf{a} \times \mathbf{b}|$.
(c) By use of the Right Hand Rule, for each component of $\mathbf{a} \times \mathbf{b}$, determine whether it is positive, negative or zero.

6. Consider the plane $2x + 3y + 4z - 12 = 0$.
(a) Find x-intercept, y-intercept and z-intercept of the plane. Use these intercepts to sketch a triangle representing the plane in the xyz-coordinate system.

(b) Write the equation of the intersection of the plane with the xy-plane? What is that intersection? Indicate it on the same picture.

7. Assume that $A = (1, 2, -1)$, $B = (2, 0, 2)$ and $C = (3, -2, 1)$.
(a) Find a unit vector perpendicular to the plane that contains three A, B and C.
(b) Find the area of the triangle ABC

8. Find the coordinates on the intersection point of two lines:
(L_1) $x = 1 + t,\ y = 2 - t\ , z = -\ 1 + 2t$
(L_2) $x = 3 - s\ , y = 1 + 2\ s\ , z = 4 - s$

9. (a) Write equation of the plane passing through P(1, $-2,3$) and perpendicular to the vector OP

(b) Write the symmetric equation of the line passing through Q$(2,4,\ -\ 5)$ and perpendicular to the plane : $3\,x + y - 2\,z + 1\,5 = 0$.

10. Find the point in which the line with parametric equations: $x = 2 - t\ , y = 1 + 3\,t\ , z = 4\,t$ intersects the plane: $2\,x - y + z = 2$

11. (a)**.** Find all values of x for which two vectors $< x\ , 1, 4 >$ and $< x\ , 3\ , x >$ are perpendicular.

(b)**.** A constant force with vector representation $\mathbf{F} = 5\boldsymbol{i} - 6\boldsymbol{j} + 10\boldsymbol{k}$ moves an object from the point $(2,3,0)$ to the point $(4,\ 9,\ 12)$. Find the work done.

12. Let **R** be the region enclosed by the graph of $y = x^3$, the x-axis and the vertical line $x = 1$. Find the volume generated by rotating **R** about the x-axis.

13. Let **R** be the region enclosed by the graph of $y = x^{\frac{1}{2}}$, the y-axis and the horizontal line $y = 1$. Find the volume generated by rotating **R** about the horizontal line $y = 1$.

14. The height of a monument 20 m. A horizontal cross-section at a distance x meters from the top is an equilateral triangle with side $x/4$ meters. Find the volume of the monument.

ANSWERS

1. (b) $<$ -2, 4, 1 $>$

2. (a) $(1,\ 0,$ -2) (b) $\sqrt{13}$ (c) $x = 1$

3. (a) $(x - 1)^2 + (y - 2)^2 + (z + 1)^2 = 16$

(b) $(x - 1)^2 + (z + 1)^2 = 12$

4. (a) $< 7,$ - 41, 8 $>$ (b) $-\ 2\ 2$ (c) $-\ \frac{22}{9}$ (d) $< -\ 44,6, -\ 3\ 3 >$

5. (a) 0 (Why?) (b) 6 (Why?)

(c) x-component > 0 , y-component < 0 and z-component $= 0$
(the vector lies in the second quadrant in the xy-plane)

6. (a) x-inetercept is 6; y-inetercept is 4; z-inetercept is 3
(b) the intersection with the xy-plane is the line whose equation is $2x + 3y - 12 = 0$

7. (a) $\frac{1}{\sqrt{5}} < 2,\ 1\ ,0 >$

(b) $2\sqrt{5}$

8. (4, -1 , 5)

9. (a) $x - 2y + 3z - 14 = 0$

(b) $x = 2 + 3t$
$y = 4 + t$
$z = -5 - 2t$

10. $(1,4,4)$

11. (a) $x = -1$ or $x = -3$ (b) 94

12. $\frac{\pi}{7}$

13. $\frac{\pi}{6}$

14. $\frac{125\sqrt{3}}{3}$ m^3